Addendum to closed orbits of fixed energy for a class of N-body problems


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Addendum to

Closed orbits of fixed energy for a class of N-body problems

by

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In Lemma of 6 the paper Closed Orbits of fixed energy for a class of N-body problems, published on this Journal (Vol. 9, No. 2, 1992, pp. 187-200) it is claimed that critical points of $f_\varepsilon$ are found by applying the Mountain-Pass Theorem. Actually, such a Theorem needs to be slightly changed, in order to make it applicable in Lemma 6. We indicate below the required modification.

Let $\Sigma_\rho = \{ u \in \Lambda_0 : \| u \| \geq \rho \}$, $\Gamma_\rho = \{ p \in C([0, 1], \Lambda_0) : \| p(0) \| = \rho, \ p(1) = u_1 \}$, and

$$c = \inf_{p \in \Gamma_\rho} \max_{0 \leq \xi \leq 1} f_\varepsilon(p(\xi)).$$

Hereafter, $0 < \varepsilon \leq \varepsilon_0$. By Lemma 2 (i), $a \geq \beta > 0$. Suppose $c$ is not a critical level for $f_\varepsilon$. Since $(PS^+)$ holds, then there exist $m > 0$ and $k \in [0, \beta/2]$ such that $\| f_\varepsilon(p) \| \geq m$, for all $u \in \Lambda_0 \cap \{ c - k \leq f_\varepsilon(u) \leq c + k \}$. Let $\eta(s, u)$, $\eta : [0, \tau_u] \times \Lambda_0 \to E$ denote the steepest descent flow satisfying

$$\frac{d\eta}{ds} = -X(\eta), \quad \eta(0, u) = u,$$
where $X$ is, as usual, a pseudo-gradient vector field for $f_\epsilon$, such that (i) $X(u) = 0$ if $f_\epsilon(u) \leq c - 2k$ or $f_\epsilon(u) \geq c + 2k$, (ii) $X(u) = f'_\epsilon$ if $c - k \leq f_\epsilon(u) = c + k$, and (iii) $f_\epsilon(\eta(s, u)) \leq f_\epsilon(u)$, $\forall 0 \leq s < \tau_u$ (see the Deformation lemma in [2]).

Since, as a consequence of (2.3), $f_\epsilon(u) \to +\infty$ whenever $u \to v \in \partial \Lambda_0 - \{0\}$, then (iii) above readily implies that $\eta(s, u) \in \Lambda_0$ whenever $u \in \Lambda_0$ and $\eta(s, u) \neq 0$.

Let us show that $\tau_u \geq \rho/2$, for all $u \in \Sigma_\rho$. According to the preceding remark, this follows in the usual way if $\|\eta(s, u)\| > \rho/2$, $\forall s < \tau_u$ (indeed, in such a case $\tau_u = +\infty$). Otherwise, let $S \in ]0, \tau_u[$ be such that $\|\eta(S, u)\| = \rho/2$, for some $u \in \Sigma_\rho$. Then

$$\frac{\rho}{2} \leq \|\eta(S, u) - u\| \leq \int_0^S \|X(\eta)\| \leq S,$$

proving the claim.

Note also that the same arguments used to prove Lemma 2 (i) yield that $(f'_\epsilon(u)|u) > 0$, $\forall u \in \Lambda_0$, $\|u\| = \rho$. Therefore $\|\eta(s, u)\| < \rho$, whenever $s \geq 0$ and $u \in \Lambda_0$, $\|u\| = \rho$.

After these preliminaries, let $k' < \min (k, \rho/4m)$ and let $p \in \Gamma_\rho$ be such that $\max f_\epsilon(p(\xi)) < c + k'$. Consider the path $p_1(\xi) = \eta\left(\frac{p}{2}, p(\xi)\right)$. Using the properties of the vector field $X$, one shows in the standard way that $\max f_\epsilon(p_1(\xi)) < c - k'$. Moreover, $p_1(1) = u_1$, whereas, as noted before, $\|p_1(0)\| < \rho$. Let $\xi_0 = \min \{\chi \in [0, 1]: \|p_1(\xi)\| > \rho \}$, $\forall \xi \in [\chi, 1]$. Setting $q(\xi) = p_1(\xi_0 + \xi(1 - \xi_0))$, it follows that $q \in \Gamma_\rho$, as well as $\max f_\epsilon(q(\xi)) < c - k'$, a contradiction which shows that $c$ is a critical level for $f_\epsilon$. 

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