ANNALES DE L'I. H. P., SECTION B

M. F. BARNSLEY
S. G. DEMKO
J. H. ELTON
J. S. GERONIMO

Erratum Invariant measures for Markov processes arising from iterated function systems with place-dependent probabilities

Annales de l'I. H. P., section B, tome 25, nº 4 (1989), p. 589-590
<http://www.numdam.org/item?id=AIHPB_1989__25_4_589_0>

Ann. Inst. Henri Poincaré,

Vol. 25, n° 4, 1989, p. 589-590. *Probabilités et Statistiques*

Erratum

Invariant measures for Markov processes arising from iterated function systems with place-dependent probabilities

by

M. F. BARNSLEY, S. G. DEMKO, J. H. ELTON and J. S. GERONIMO

Ann. Inst. Henri Poincaré, vol. **24**, n° 3, 1988, p. 367-394

The proof given for Lemma 2.5, pg. 374, while correct for certain spaces, *e.g.* $\mathscr{R}^1$, is incorrect in general, as it assumes special properties of the modulus of continuity. A correct proof is obtained by replacing from the beginning of the proof through lines 13, page 375, by the following:

Proof. — Note that each φ_i is non-decreasing, and $\varphi_i(t) \leqq 1$ for all t, since $|p_i(x) - p_i(y)| \leqq 1$ for all x, y. Let

$$\varphi_0(t) = \begin{cases} t, & 0 \leqq t \leqq 1 \\ 1, & t > 1. \end{cases}$$

Let $\varphi = \varphi_0 \vee \varphi_1 \vee \ldots \vee \varphi_N$, where $t \vee u$ denotes $\max\{t, u\}$. It is clear that φ also satisfies Dini's condition.

Sublemma. — Let $\varphi : [0, 1] \to [0, \infty)$ be non-decreasing, with

$$\int_0^1 \frac{\varphi(t)}{t} dt < \infty.$$

Then there exists $\psi : [0, 1] \to [0, \infty)$ such that $\psi(t) \geqq \varphi(t)$ for all t, $\dfrac{\psi(t)}{t}$ is non-increasing, and

$$\int_0^1 \frac{\psi\ t)}{t} dt < \infty.$$

Proof. — W.L.O.G. assume $\varphi(^+) = \varphi(t)$, $\forall t$. Let $f(t) = \varphi(t)/t$. We shall use the "rising sun" lemma of F. Riesz (Boas, page 134): Let E be the "shadow region" for the sun rising in the direction of the positive x-axis; that is, $E = \{t \in (0, 1) : \exists x > t \text{ with } f(x) > f(t)\}$. Then E is an open set and if (a, b) is any one of the open intervals comprising E, $f(x) \leqq f(b)$ for $x \in (a, b)$, and $f(a) = f(b)$ since f is right-continuous and has only upward jumps.

Let C be countable collection of non-overlapping open intervals such that $E = \bigcup C$. Define

$$g(x) = \begin{cases} f(x), & x \notin E \\ f(b), & x \in (a, b) \in C. \end{cases}$$

Thus $g(x)$ is non-increasing and $g \geqq f$. Now if $(a, b) \in C$,

$$\cdot \int_a^b [g(x) - f(x)]\,dx = \int_1^b [f(b) - f(x)]\,dx$$
$$= \int_a^b \left[\frac{\varphi(b)}{b} - \frac{\varphi(x)}{x}\right] dx \leqq \int_a^b \left[\frac{\varphi(b)}{b} - \frac{\varphi(a)}{b}\right] dx$$

since φ is non-decreasing. Thus

$$\int_a^b [g(x) - f(x)]\,dx \leqq [\varphi(b) - \varphi(a)]\frac{b-a}{b} \leqq \varphi(b) - \varphi(a),$$

so

$$\int_0^1 [g(x) - f(x)]\,dx = \int_E [g(x) - f(x)]\,dx$$
$$= \sum_{(a,\, b) \in C} \int_a^b [g(x) - f(x)]\,dx \leqq \sum_{(a,\, b) \in C} \varphi(b) - \varphi(a) \leqq \varphi\, 1)$$

since φ is non-decreasing, so $\int_0^1 g(t)\,dt < \infty$. Finally, let

$$\psi(t) = tg(t) \geqq tf(t) = \varphi(t), \qquad \text{and} \qquad \frac{\psi(t)}{t} = g(t)$$

is non-increasing. □

So by the sublemma, increasing φ if necessary, we may assume in what follows that $\varphi(t)/t$ is non-increasing and φ is Dini.

Let $f \in C_c(X)$, $\| f \| \leqq 1$, and also assume $f \in \mathrm{Lip}_1$, so

$$| f(x) - f(y) | \leqq C\, d(x, y), \qquad \forall x, y \in X.$$

We may take $C \geqq 2$.

Without loss of generality, we may assume $q \leqq 1$ in the hypothesis of the lemma.

Define

$$\beta^*(t) = \frac{N \vee C}{1 - r^q} \int_0^{tr^{-q}} \frac{\varphi(u)}{u}\,du.$$

This is finite since φ is Dini. Then $\beta^*(0) = 0$, and β^* is continuous and strictly increasing. Also, β^* is a *concave* function, since $\varphi(t)/t$ is non-increasing.

We thank Roger Nussbaum for pointing out that our earlier proof was incorrect.

REFERENCES

R. P. Boas, *A Primer of Real Functions*, Wiley, 1970.