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Erratum

Invariant measures for Markov processes arising from iterated function systems with place-dependent probabilities

by

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The proof given for Lemma 2.5, pg. 374, while correct for certain spaces, e.g. $\mathbb{R}^1$, is incorrect in general, as it assumes special properties of the modulus of continuity. A correct proof is obtained by replacing from the beginning of the proof through lines 13, page 375, by the following:

**Proof.** Note that each $\varphi_i$ is non-decreasing, and $\varphi_i(t) \leq 1$ for all $t$, since $|p_i(x) - p_i(y)| \leq 1$ for all $x, y$. Let

$$
\varphi_0(t) = \begin{cases} 
t, & 0 \leq t \leq 1 \\
1, & t > 1.
\end{cases}
$$

Let $\varphi = \varphi_0 \vee \varphi_1 \vee \ldots \vee \varphi_N$, where $t \vee u$ denotes $\max\{t, u\}$. It is clear that $\varphi$ also satisfies Dini’s condition.

**Sublemma.** Let $\varphi : [0, 1] \to [0, \infty)$ be non-decreasing, with

$$
\int_0^1 \frac{\varphi(t)}{t} \, dt < \infty.
$$

Then there exists $\psi : [0, 1] \to [0, \infty)$ such that $\psi(t) \geq \varphi(t)$ for all $t$, $\frac{\psi(t)}{t}$ is non-increasing, and

$$
\int_0^1 \frac{\psi(t)}{t} \, dt < \infty.
$$

**Proof.** W.L.O.G. assume $\varphi(0) = \varphi(t), \forall t$. Let $f(t) = \varphi(t)/t$. We shall use the “rising sun” lemma of F. Riesz (Boas, page 134): Let $E$ be the “shadow region” for the sun rising in the direction of the positive $x$-axis; that is, $E = \{t \in (0, 1) : \exists x > t \text{ with } f(x) > f(t)\}$. Then $E$ is an open set and if $(a, b)$ is any one of the open intervals comprising $E$, $f(x) \leq f(b)$ for $x \in (a, b)$, and $f(a) = f(b)$ since $f$ is right-continuous and has only upward jumps.

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Let $C$ be countable collection of non-overlapping open intervals such that $E = \bigcup C$. Define

$$ g(x) = \begin{cases} f(x), & x \notin E \\ f(b), & x \in (a, b) \in C. \end{cases} $$

Thus $g(x)$ is non-increasing and $g \geq f$. Now if $(a, b) \in C$,

$$ \int_a^b [g(x) - f(x)] \, dx = \int_a^b [f(b) - f(x)] \, dx $$

$$ = \int_a^b \left[ \frac{\varphi(b) - \varphi(x)}{b} - \frac{\varphi(a)}{x} \right] \, dx \leq \int_a^b \left[ \frac{\varphi(b)}{b} - \frac{\varphi(a)}{b} \right] \, dx $$

since $\varphi$ is non-decreasing. Thus

$$ \int_a^b [g(x) - f(x)] \, dx \leq [\varphi(b) - \varphi(a)] \frac{b-a}{b} \leq \varphi(b) - \varphi(a), $$

so

$$ \int_a^b \frac{[\varphi(b) - \varphi(a)]}{b} \, dx \leq \sum_{(a, b) \in C} \frac{\varphi(b) - \varphi(a)}{b} \leq \varphi(1) $$

since $\varphi$ is non-decreasing, so $\int_0^1 g(t) \, dt < \infty$. Finally, let

$$ \psi(t) = tg(t) \geq tf(t) = \varphi(t), \quad \text{and} \quad \psi(t) = g(t) $$

is non-increasing. $\square$

So by the sublemma, increasing $\varphi$ if necessary, we may assume in what follows that $\varphi(t)/t$ is non-increasing and $\varphi$ is Dini.

Let $f \in C_c(X)$, $\|f\| \leq 1$, and also assume $f \in \text{Lip}_1$, so

$$ |f(x) - f(y)| \leq C d(x, y), \quad \forall x, y \in X. $$

We may take $C \geq 2$.

Without loss of generality, we may assume $q \leq 1$ in the hypothesis of the lemma.

Define

$$ \beta^*(t) = \frac{N \vee C}{1 - t^q} \int_0^{u - t^{-q}} \frac{\varphi(u)}{u} \, du. $$

This is finite since $\varphi$ is Dini. Then $\beta^*(0) = 0$, and $\beta^*$ is continuous and strictly increasing. Also, $\beta^*$ is a concave function, since $\varphi(t)/t$ is non-increasing.

We thank Roger Nussbaum for pointing out that our earlier proof was incorrect.

REFERENCES