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Correction to « Domains of attraction in several dimensions »

by

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For $p > 2$ let X be a l_p -valued random variable defined as follows:

$$(1) \quad X = \varepsilon \sum_{N^2 < k \leq N^2 + N} e_k$$

where $\{e_k\}$ is the canonical basis of l_p , ε is a Bernoulli random variable with parameter $1/2$ and N is a random variable independent of ε and with distribution

$$\begin{aligned} P\{N = n\} &= c/n^{1+2/p} \text{ for } n \in [2, 2^2) \cup [2^{2^2}, 2^{2^3}) \cup \dots \cup [2^{2^{2^k}}, 2^{2^{2^{k+1}}}) \cup \dots \\ P\{N = n\} &= 0 \quad \text{otherwise.} \end{aligned}$$

In example 3.5 of [2] it is claimed that X is in the domain of partial attraction of only one law (one type of laws), the centered Gaussian law γ with the covariance of X , which exists, but that nevertheless X is not in its domain of attraction. To the best of our knowledge this statement is true, however the proof given in [2] of the fact that X is actually in the domain of partial attraction of γ is not correct. In this note we give a (hopefully) correct proof of this.

The notation will be as in [2]; in particular, if $x \in l_p$, $x_{(k)}$ will denote its k -th coordinate in the canonical basis.

We need the following

PROPOSITION. — Assume that X is a l_p -valued symmetric random variable.

$p \geq 2$. Let $\{X_i\}$ be independent identically distributed copies of X , and let $n_k \uparrow \infty$, $n_k \in \mathbb{N}$. Then, in order that the sequence

$$(2) \quad \left\{ \mathcal{L}(\Sigma_{i=1}^{n_k} X_i / n_k^{1/2}) \right\}_{k=1}^{\infty}$$

converge weakly to a Gaussian law it is (necessary and) sufficient that the following conditions hold:

- i) X is pregaussian,
- ii) for every $\delta > 0$, $\lim_{k \rightarrow \infty} n_k \mathbf{P} \{ \|X\| > \delta n_k^{1/2} \} = 0$,
- iii) $\lim_{R \rightarrow \infty} \sup_k n_k^{1-p/2} \Sigma_{m=R+1}^{\infty} \mathbf{E} |X_{(m)}|^p \mathbf{I}_{[\|X\| \leq n_k^{1/2}]} = 0$.

This proposition follows directly from Theorem 3.1 in [3]. See also exercises 13 and 17, p. 205-206, in [1].

We will prove that the variable X defined in (1) satisfies conditions (i)-(iii) of the previous proposition for the sequence $\{n_k\}$ given by

$$(3) \quad n_k = [2^{2^{2k} + 3/2(2/p)}], \quad k \in \mathbb{N}$$

where $[\]$ denotes « integer part of ».

It is already proved in [2] that X is pregaussian. So we need only prove that X verifies properties (ii) and (iii).

Proof of ii). — Obviously, given $\delta > 0$ there exists k_δ such that for $k > k_\delta$

$$2^{2^{2k} + 1} < \delta^p n_k^{p/2}.$$

Since $\|X\| = N^{1/p}$ it follows that for $k > k_\delta$,

$$\begin{aligned} n_k \mathbf{P} \{ \|X\| > \delta n_k^{1/2} \} &= n_k \mathbf{P} \{ N > \delta^p n_k^{p/2} \} \\ &\leq c 2^{2^{2k} + 3/2(2/p)} \Sigma_{n=2^{2^{2k}+2n}^{-1-2/p}}^{\infty} \approx 2^{-1} c p 2^{2^{2k}(2/p)(2^{3/2}-4)} \\ &\rightarrow 0 \quad \text{as } k \rightarrow \infty. \end{aligned}$$

Proof of iii). — Let us first observe that

$$\begin{aligned} (4) \quad \Sigma_{m=R+1}^{\infty} \mathbf{E} |X_{(m)}|^p \mathbf{I}_{[\|X\| \leq n_k^{1/2}]} &= \Sigma_{m=R+1}^{\infty} \mathbf{P} \{ N^2 < m \leq N^2 + N, N \leq n_k^{p/2} \} \\ &\leq \Sigma_{s=[R^{1/2}]}^{\infty} s \mathbf{P} \{ N = s, N \leq n_k^{p/2} \} = \begin{cases} 0 & \text{if } [R^{1/2}] > n_k^{p/2} \\ \Sigma_{s=[R^{1/2}]}^{n_k^{p/2}} s \mathbf{P} \{ N = s \} & \text{otherwise.} \end{cases} \end{aligned}$$

Let $r \in \mathbb{N}$ be such that $2^{2r} \leq [R^{1/2}] < 2^{2r+2}$. By (4) the limit in (iii) is then bounded above by

$$\begin{aligned} \lim_{R \rightarrow \infty} \sup_{k \geq r} c n_k^{1-p/2} \int_{[2^{2^{2r}-1}, 2^{2^{2r+1}}] \cup \dots \cup [2^{2^{2k}-1}, 2^{2^{2k+1}}]} x^{-2/p} dx \\ \leq \lim_{r \rightarrow \infty} \sup_{k \geq r} c (1 - 2/p)^{-1} n_k^{1-p/2} k 2^{2^{2k} + 1(1-2/p)} \\ = \lim_{r \rightarrow \infty} c (1 - 2/p)^{-1} r 2^{2^{2r} + 1(1-2/p)(1-2^{1/2})} = 0. \end{aligned}$$

The proof that X satisfies (ii) and (iii) is thus completed. Hence, X is in the domain of partial attraction of the centered Gaussian law which has its covariance.

There is a trivial error in [2] which we also correct now. The last two lines of the proof of Theorem 3.1, part (2) should read: « ... therefore, if $\mu \{ \|x\| > 0 \} = 0$ we have $P \{ \|X\| > a_{n_i} \} / a_{n_i}^{-2} E \|X\|^2 I_{\{\|X\| \leq a_{n_i}\}} \rightarrow 0$ as ... ».

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