HELLMUT BAUMGÄRTEL

On the structure of relative identification operators for quantum fields and their connection with the Haag-Ruelle scattering theory


<http://www.numdam.org/item?id=AIHPA_1984__40_3_225_0>

© Gauthier-Villars, 1984, tous droits réservés.

L’accès aux archives de la revue « Annales de l’I. H. P., section A » implique l’accord avec les conditions générales d’utilisation (http://www.numdam.org/conditions). Toute utilisation commerciale ou impression systématique est constitutive d’une infraction pénale. Toute copie ou impression de ce fichier doit contenir la présente mention de copyright.
On the structure of relative identification operators for quantum fields and their connection with the Haag-Ruelle scattering theory

by

Hellmut BAUMGÄRTEL
Institut für Mathematik,
Akademie der Wissenschaften der DDR, Berlin

ABSTRACT. — The paper recalls the notion of a relative identification operator $K$ of a Wightman field $A(\cdot)$ with respect to a corresponding free field $A^0(\cdot)$, useful for the definition of wave operators with respect to the field $A(\cdot)$. The corresponding pre-wave operator $e^{itH}Ke^{-itH_0}$ can be linked directly with the Haag-Ruelle approximants of the field $A(\cdot)$. Thus the Haag-Ruelle scattering theory can be embedded formally into the framework of the abstract scattering theory. Some structural properties of $K$ are presented. It is pointed out that $K$ is uniquely determined by a single field operator $A(h(p)\gamma_0(p))$, where $h(p)$ is a smooth function with support in a sufficiently small neighbourhood of the discrete mass hyperboloid characterized by the mass $m_0 > 0$ belonging to $A^0(\cdot)$ (as is usually introduced within the Haag-Ruelle framework) and where $\gamma_0 \in \mathcal{S}(\mathbb{R}^3)$ is multiplicative-generating.

RÉSUMÉ. — On rappelle la notion d’opérateur d’identification relatif $K$ d’un champ de Wightman $A(\cdot)$ par rapport au champ libre correspondant $A^0(\cdot)$, utile pour la définition des opérateurs d’onde du champ $A(\cdot)$. Le pré-opérateur d’onde $e^{itH}Ke^{-itH_0}$ correspondant peut être relié directement aux approximants de Haag-Ruelle pour le champ $A(\cdot)$. Ainsi la théorie de la diffusion de Haag-Ruelle peut être formellement incorporée dans le cadre de la théorie abstraite de la diffusion. On donne quelques propriétés structurelles de $K$. On remarque que $K$ est déterminé de façon unique par un seul opérateur de champ $A(h(p)\gamma_0(p))$, où $h(p)$ est une fonction lisse à support contenu dans un voisinage assez petit de l’hyperboloiade de
mass caractérisé par la masse $m_0 > 0$ du champ $A_0(\cdot)$ (comme on l'introduit de façon usuelle dans la théorie de Haag-Ruelle), et où $\gamma_0 \in \mathcal{S}(\mathbb{R}^3)$ est génératrice par multiplication (voir Définition 3).

§ 1. INTRODUCTION

Let $A(\cdot)$ be a Wightman field on a (separable) Hilbert space $\mathcal{H}$. For convenience we collect the properties of such a field. The tensor algebra over the Schwartz space $\mathcal{S}(\mathbb{R}^4)$ is denoted by $\mathcal{T}$. Its elements are finite sequences $f := \{ f_0, f_1, \ldots, f_N, 0, \ldots \}$, where $N$ depends on $f$ and where $f_n \in \mathcal{S}(\mathbb{R}^{4n})$. $\mathcal{T}$ is equipped with the usual topology $\tau$ (locally convex direct sum of the Schwartz space topologies of the $\mathcal{S}(\mathbb{R}^{4n})$). The Wightman functional $W(\cdot) : \mathcal{T} \mapsto \mathbb{C}$ of the field $A(\cdot)$ is assumed to be linear, normed, positive, continuous and Poincaré invariant. There is a unique vacuum $\omega \in \mathcal{H}$ and the field is assumed to be spectral and local. The continuous linear functionals on $\mathcal{F}$ have the form $W = \{ W_0, W_1, W_2, \ldots \}$, where $W_n$ is a continuous linear functional on $\mathcal{S}(\mathbb{R}^{4n})$, a so-called $n$-point functional, and $W(f) = \Sigma W_n(f_n)$. Recall the special form of the functionals

\begin{align}
(1) & \quad W_0(f_0) = f_0, \\
(2) & \quad W_1(f_1) = \gamma f_1(0), \quad \gamma \in \mathbb{R}, \\
(3) & \quad W_2(f_2) = \gamma f_2(0, 0) + \int_0^\infty \int_{H_m} f_2(-p, p) \mu_m(dp) \rho(dm).
\end{align}

Note that we prefer to work with momentum coordinates, that is with functions $f_n(p_1, p_2, \ldots, p_n) \in \mathcal{S}(\mathbb{R}^{4n})$ which are Fourier transforms

$$ f_n(p_1, \ldots, p_n) = (2\pi)^{-2n} \int e^{-\frac{1}{2} \sum_{j=1}^n (p_{j}, x_j)} f_n(x_1, \ldots, x_n) dx_1 \ldots dx_n $$

of functions $\tilde{f}_n \in \mathcal{S}(\mathbb{R}^{4n})$ depending on position coordinates. $(\cdot, \cdot)$ denotes the Cartesian scalar product in $\mathbb{R}^4$. $H_m$ denotes the mass hyperboloid $H_m := \{ p : p_0^2 - |p|^2 = m^2, p_0 > 0 \}$, $\mu_m(\cdot)$ denotes the Lorentz invariant measure on $H_m$ given by $\mu_m(dp) = dp/(m^2 + |p|^2)^{1/2}$ and $\rho(\cdot)$ is a characteristic polynomially bounded Borel measure on $m \geq 0$. Formula (3) is called the Källen-Lehmann representation of the 2-point functional.

The set of all $f \in \mathcal{F}$ satisfying $W(f^* f) = 0$ is denoted by $\text{ker } W$ (left kernel). Note that $f^*$ denotes the usual conjugation in $\mathcal{F}$, $f_n(p_1, \ldots, p_n) = \tilde{f}_n(-p_{m}, -p_{n-1}, \ldots, -p_1)$.

The field $A(\cdot)$ is defined on $\mathcal{F}$, i.e. $A(f), f \in \mathcal{F}$, is a generalized field.
operator, the usual field operator is given by \( A(f_i) \) where \( f_i = (0, f_1, 0, \ldots) \in \mathcal{F} \). For brevity we write also \( A(f_i) \) in this case.

Furthermore, an upper and lower mass gap is assumed, the discrete mass is denoted by \( m_0 \), the corresponding one-particle subspace of \( \mathcal{H} \) is denoted by \( \mathcal{H}_1 \), it is assumed to be irreducible with respect to the Poincaré group, the corresponding representation is labeled by \( m_0 \) and \( s = 0 \). Recall the representation (SNAG-theorem)

\[
U_a = \int_{\mathbb{R}^4} e^{-i(a, p)} E(dp)
\]

for the unitary representation \( U_a \) of the translation group \( a \in \mathbb{R}^4 \) associated with the field. In terms of \( E(\cdot) \) the mass gap is expressed by

\[
\text{supp}^m E = \{ 0 \} \cup \{ m_0 \} \cup \Lambda, \Lambda \subseteq [m_0 + \varepsilon, \infty), \varepsilon > 0, m_0 > 0,
\]

where \( \text{supp}^m E \) denotes the mass spectrum (note that \( \text{supp} E \subseteq \text{clo} V^+ \), \( V^+ \) the forward cone, and that \( \text{supp} E \) is Lorentz invariant, i.e. it contains only full mass hyperboloids \( H_m \), then the mass spectrum is the closure of all \( m \) such that \( H_m \subseteq \text{supp} E \).

Finally, the condition of « coupling of the vacuum to the one-particle states » is assumed to be satisfied (see for example M. Reed and B. Simon [1, p. 319]. Note that this condition is satisfied if and only if

\[
m_0 \in \text{supp} \rho
\]

is valid. That is, in this case one obtains

\[
\{ m_0 \} \subseteq \text{supp} \rho \subseteq \text{supp}^m E,
\]

(the latter inclusion is obvious).

The free (scalar) field, corresponding to \( m_0 > 0 \) and \( s = 0 \), is denoted by \( A^0(\cdot) \), acting on the Hilbert space \( \mathcal{H}^0 \). Its measure « \( \rho \) » (mass distribution of the Källen-Lehmann representation) is given by the Dirac measure \( \rho(m) = \delta(m - m_0) \).

In H. Baumgärtel et al. [2] a so-called relative identification operator \( K \) is introduced, useful for the definition of wave operators with respect to the field \( A(\cdot) \). For convenience, we recall the definition and simple properties of \( K: \mathcal{H}^0 \leftrightarrow \mathcal{H} \). First, by \( h \in C_{\text{c}}(\mathbb{R}^4) \) we denote a fixed real-valued function, \( 0 \leq h \leq 1 \), with the following properties:

\[
i) \text{supp } h \subseteq \bigcup_{m_0 - \delta, m_0 + \delta} H_m, \delta > 0,
\]

\[
ii) h(\Lambda p) = h(p) \quad \text{for all } \Lambda \in \mathcal{L}_+ (\text{proper Lorentz group}),
\]

\[
iii) h \upharpoonright H_{m_0} = 1.
\]

Second, we define a certain linear manifold \( \mathcal{L} \subseteq \mathcal{F} \) by: \( g \in \mathcal{L} \) if and only if \( g_0 \in \mathbb{C} \) arbitrary, \( g_n(p_1, p_2, \ldots, p_n) = h(p_1) h(p_2) \ldots h(p_n) \gamma_n(p_1, p_2, \ldots, p_n) \), \( \gamma_n \in \mathcal{F}(\mathbb{R}^{3n}), \gamma_n \) symmetric with respect to \( p_1, p_2, \ldots, p_n \).
Then, if \( (W^0(\cdot)) \) denotes the Wightman functional and \( J^0 \) denotes the (absolute) identification operator of the free field, it turns out that \( \mathcal{L} \) contains exactly one element from each equivalence class mod \( \ker W^0 \), that is, \( \text{ima}(J^0 \mid \mathcal{L}) = \text{ima} J^0 \) and: \( f \in \mathcal{L} \) and \( J^0 f = 0 \) imply \( f = 0 \), in other words, \( \mathcal{L} \cap \ker W^0 = \{ 0 \} \) and \( \mathcal{L} \oplus \ker W^0 = \mathcal{H} \). Now an operator \( K: \mathcal{H}^0 \rightarrow \mathcal{H} \) can be defined by

\[
K \{ A^0(f) \omega^0 \} := A(f) \omega, \quad f \in \mathcal{L},
\]

or

\[
K \{ J^0 f \} := Jf, \quad f \in \mathcal{L},
\]

where \( J \) denotes the (absolute) identification operator of the field \( A(\cdot) \). (9) means that \( K \) is a certain factorization of \( J, J = KJ^0 \), on \( \mathcal{L} \). \( K \) is called the relative identification operator between \( A^0(\cdot) \) and \( A(\cdot) \) with respect to \( \mathcal{L} \). Recall the following simple properties of \( K \):

I) \( K \) is densely defined, \( \text{dom} \ K = \text{ima} J^0 \), which is dense in \( \mathcal{H}^0 \).

II) \( \text{dom} \ K \) is invariant with respect to \( U_g^0 \) (the unitary representation of the Poincaré group belonging to the free field).

III) \( K \) is continuous with respect to the Schwartz space topology \( \tau \) of \( \mathcal{F} \) (more precisely: \( \text{dom} \ K \) may be equipped with this topology by the bijection \( \mathcal{L} \ni f \mapsto J^0 f \in \text{dom} \ K \), then \( K \) is continuous with respect to this topology).

IV) \( K \omega^0 = \omega \).

V) The intertwining relation

\[
U_g K = K U_g^0
\]

is valid if \( g = \{ \Lambda_0, (0, a) \} \), where \( a \in \mathbb{R}^3 \) is a pure spatial translation and where \( \Lambda_0 \) is a pure rotation in the a-space (the intertwining relation (10) is not valid in general for time translations).

Using \( K \), the standard two-space pre-wave operator is given by

\[
e^{i t H} K e^{-i t H^0} u, \quad u \in \text{dom} \ K,
\]

where \( e^{-i t H} = u_{(t,0)}, \ e^{-i t H^0} = U^0_{(t,0)} \) denote the unitary representations of the time translations in \( \mathcal{H}, \mathcal{H}^0 \), respectively.

In this paper some further structural properties of \( K \) are presented. In fact, it is shown that the expression (11) is intimately connected with the Haag-Ruelle approximants with respect to the field \( A(\cdot) \).

If \( \mathcal{M} \) is a subset of \( \mathcal{F} \), for brevity we denote by \( \mathcal{M}^{(n)} \) the set of all \( f \in \mathcal{M} \) with \( f = \{ 0, \ldots, 0, f_m, 0, \ldots \} \), i. e. the intersection of \( \mathcal{M} \) with \( \mathcal{F}(\mathbb{R}^{4n}) \).

Finally recall the assignment between one-particle states and field operators.

If \( f \in \mathcal{L}^{(1)} \) then \( A(f) \omega \in \mathcal{H}_1 \). Moreover, the assignment \( \mathcal{L}^{(1)} \ni f \mapsto A(f) \omega \in \mathcal{H}_1 \) is an injection, the image \( \{ A(f) \omega, f \in \mathcal{L}^{(1)} \} \) coincides with

\[
\{ A(f) \omega, f \in \mathcal{F}^{(1)}, \supp f \cap \supp E \subseteq H_{m_0} \}
\]

and this linear manifold is dense in \( \mathcal{H}_1 \).

Annales de l'Institut Henri Poincaré - Physique théorique
That is, the vector \( u = A(f) \omega \in \mathcal{H}_1 \) with \( f \in \mathcal{L}^{(1)} \) is in one-to-one correspondence with \( f \). Therefore, one has an assignment of one-particle states \( u \in \mathcal{H}_1 \) to certain field operators \( B_u = A(f) \). This assignment satisfies the property \( B_u \omega = u \).

On the other hand, the assignment can be considered as the assignment of vectors of the free one-particle space to vectors \( u \in \mathcal{H}_1 \) via \( K \). Namely, if \( f \in \mathcal{L}^{(1)} \), then \( A^0(f) \omega^0 = \{ 0, \gamma(p), 0, \ldots \} \), where \( f_1(p) = h(p) \gamma(p) \) and where \( \gamma(p) \) is to be considered as an element of \( L^2(\mathbb{R}^3, dp/\mu(p)) \). That is, we have

\[
\mathcal{H}^0_1 = L^2(\mathbb{R}^3, dp/\mu(p)) \ni \gamma(p) \mapsto A(f) \omega = u \in \mathcal{H}_1,
\]

where \( \mu(p) := (m_0^2 + |p|^2)^{1/2} \).

\section{2. Calculation of \( K \)}

In this paragraph we calculate \( K \) on a dense subdomain of \( \text{dom } K = \text{ima } J^0 = \text{ima } (J^0 \uparrow \mathcal{L}) \). For this purpose we use the structure of \( J^0 \), which is given in [2]. According to this paper (Corollary 1) we have

\[
J^0 f = \{ f_0, (Tf)_1 \uparrow H_{m_0}, S_2(Tf)_2 \uparrow H_{m_0} \times H_{m_0}, \ldots \}, \quad f \in \mathcal{F},
\]

where \( T \) denotes a certain continuous linear operator, acting in \( \mathcal{F} \) (see [2, Theorem 2]) and where \( S_n \) denotes the symmetrization operator

\[
(S_n f_n(p_1, \ldots, p_n) = (n!)^{-1} \sum_{\pi} f_n(p_{\pi(1)}, p_{\pi(2)}, \ldots, p_{\pi(n)}).
\]

Note that \( Tf = f \) for \( f \in \mathcal{L} \) (cf. [2, Lemma 5]), that is, one obtains

\[
J^0 f = \{ f_0, f_1 \uparrow H_{m_0}, S_2 f_2 \uparrow H_{m_0} \times H_{m_0}, \ldots \}, \quad f \in \mathcal{L}.
\]

Recall that \( f \in \mathcal{L} \) means \( f_n(p_1, \ldots, p_n) = \prod_{j=1}^{n} h(p_j) \gamma_n(p_1, p_2, \ldots, p_n) \), where \( \gamma_n \) is symmetric. Therefore

\[
J^0 f = \{ f_0, \gamma_1(p_1), \gamma_2(p_1, p_2), \ldots \}, \quad f \in \mathcal{L},
\]

where \( \gamma_n(p_1, \ldots, p_n) \) is to be considered as an element of

\[
L^2(\mathbb{R}^{3n}, \bigotimes_{j=1}^{n} dp_j/\mu(p_j)),
\]

where \( \mu(p) := (m_0^2 + |p|^2)^{1/2} \).

Now let \( K_n \) be the \( n \)-particle component of \( K \), that is

\[
K_n := K \uparrow S_n(\mathcal{H}_1 \otimes \mathcal{H}_1 \otimes \ldots \otimes \mathcal{H}_1),
\]

Vol. 40, n° 3-1984
such that
\[ K u = \sum_{n=0}^{\infty} K_n u_n, \quad u = J^0 f, \quad f \in \mathcal{L}, \quad u_n = J^0 f_n. \]

Furthermore, let
\[ \gamma_n(p_1, \ldots, p_n) = S_n(\alpha_1(p_1) \otimes \alpha_2(p_2) \otimes \ldots \otimes \alpha_n(p_n)), \quad \alpha_j \in \mathcal{S}(\mathbb{R}^3). \]

According to (8) we obtain
\[ (12) \quad \gamma_n(p_1, \ldots, p_n) = S_n(\alpha_1 \otimes \alpha_2 \otimes \ldots \otimes \alpha_n) \]
\[ = S_n \left\{ \prod_{j=1}^{n} A(h(p)\alpha_j(p)) \right\} \omega, \quad n = 1, 2, \ldots, \]

where we write for brevity \( A(h(p)\alpha_j(p)) \) instead of \( A(\{0, h(p)\alpha_j(p), 0, \ldots\}) \).

We denote the linear submanifold of \( \mathcal{L} \) defined by (12) by \( \mathcal{L}_0 \subset \mathcal{L} \). \( \mathcal{L}_0 \) is dense in \( \mathcal{L} \) with respect to \( \tau \). Then we have

**Proposition 1.** The relative identification operator \( K : \mathcal{H}^0 \rightarrow \mathcal{H} \) is given on the (dense) submanifold \( \text{ima} (J^0 \uparrow \mathcal{L}_0) \) of \( \text{dom} K \) by formula (13).

**Proof:** Obvious by the preceding arguments of this paragraph. \( \blacksquare \)

According to (13), \( K \uparrow \text{ima} (J^0 \uparrow \mathcal{L}_0) \), hence \( K \) itself, is already uniquely determined by the field operators \( A(h(p)\alpha(p)), \alpha \in \mathcal{S}(\mathbb{R}^3) \). But the Poincaré covariance property of the field \( A(\cdot) \) implies a strong connection between these field operators. Namely, we have

**Lemma 2.** Let \( \alpha \in \mathcal{S}(\mathbb{R}^3), \quad f \in \mathcal{S}(\mathbb{R}^4) \). Then
\[ (14) \quad A(\hat{\alpha}(p)\hat{f}(p)) = (2\pi)^{-3/2} \int_{\mathbb{R}^3} \alpha(a)U_a A(f)U_{-a} da, \]

where \( \hat{\alpha} \) denotes the spatial Fourier transform of \( \alpha \) and \( \hat{f} \) denotes the 4-dimensional Fourier transform of \( f \). The integral on the right hand side is weakly convergent for vectors \( u, v \in \mathcal{D} = \text{ima} J \) (domain of the field operators \( A(\cdot) \)).

**Proof:** From the covariance property of \( A(\cdot) \) we obtain
\[ A(V_a f) = U_a A(f)U_{-a}, \]

where, as usual, \( (V_a f)(x) = f(x_0, x - a) \). Further we obtain
\[ A\left( \int_{\mathbb{R}^3} \alpha(a)V_a f \, da \right) = \int_{\mathbb{R}^3} \alpha(a)U_a A(f)U_{-a} da. \]
But \((V_{\alpha} f)^\wedge (p) = e^{-i(\alpha \cdot p)} \hat{f}(p)\) and
\[
\int_{\mathbb{R}^3} \alpha(a)(V_{\alpha} f)^\wedge (p) da = \int_{\mathbb{R}^3} \alpha(a)e^{-i(\alpha \cdot p)} d\tilde{a}(p) \hat{f}(p) = (2\pi)^{3/2} \tilde{a}(p) \hat{f}(p).
\]

This concludes the proof. 

The relation (14) implies that the field operators \(A(h(p)\alpha(p))\) are uniquely determined by a single field operator \(A(h(p)\gamma_0(p))\), where \(\gamma_0\) is multiplicative-generating in \(\mathcal{S}(\mathbb{R}^3)\) (together with the representation \(U_a\)).

**Definition 3.** — The function \(\gamma_0 \in \mathcal{S}(\mathbb{R}^3)\) is called multiplicative-generating with respect to \(\mathcal{S}(\mathbb{R}^3)\), if \(\{ \alpha \gamma_0 : \alpha \in \mathcal{S}(\mathbb{R}^3) \}\) is dense in \(\mathcal{S}(\mathbb{R}^3)\).

For example, if \(\gamma_0 \in \mathcal{S}(\mathbb{R}^3)\) and \(\gamma(p) \neq 0\) for all \(p \in \mathbb{R}^3\), then \(\gamma_0\) is multiplicative-generating. For example let \(\gamma_0(p) = \exp(-|p|^2)\).

**Proposition 4.** — Let \(\gamma_0 \in \mathcal{S}(\mathbb{R}^3)\) be multiplicative-generating. Then \(\{ \alpha \gamma_0 : \alpha \in \mathcal{S}(\mathbb{R}^3) \}\) is dense in \(\mathcal{S}(\mathbb{R}^3)\) and
\[
A(h(p)\tilde{a}(p)\gamma_0(p)) = (2\pi)^{-3/2} \int_{\mathbb{R}^3} \alpha(a)U_aA(h(p)\gamma_0(p))U_{-a}da.
\]

**Proof.** Obvious.

Proposition 1 and Proposition 4 together lead to an explicit description of \(K\), showing, that \(K\) is uniquely determined by \(A(h(p)\gamma_0(p))\) for some multiplicative-generating function \(\gamma_0\).

**Corollary 5.** — Let \(\gamma_0 \in \mathcal{S}(\mathbb{R}^3)\) be multiplicative-generating and put \(B_0 := A(h(p)\gamma_0(p))\). Then \(K \uparrow \text{ima}(I^0 \uparrow \mathcal{L}_0)\) is explicitly given by
\[
K_n \{ S_n(\tilde{a}_1 \gamma_0 \otimes \tilde{a}_2 \gamma_0 \otimes \ldots \otimes \tilde{a}_n \gamma_0) \}
\]
\[
= S_n \prod_{j=1}^n \left\{ (2\pi)^{-3/2} \int_{\mathbb{R}^3} \alpha_j(a)U_aB_0U_{-a}da \right\} \omega, \quad n = 1, 2, \ldots
\]

**Proof.** Obvious.

§ 3. CALCULATION OF THE PRE-WAVE OPERATOR

Let \(\gamma_0 \in \mathcal{S}(\mathbb{R}^3)\) be multiplicative-generating and put \(B_0 := A(h(p)\gamma_0(p))\). The next Proposition calculates the pre-wave operator (11).

**Proposition 6.** — Let \(\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathcal{S}(\mathbb{R}^3)\). Then
\[
(e^{iH_0}K_0 e^{-iH_0}) S_n(\tilde{a}_1 \gamma_0 \otimes \ldots \otimes \tilde{a}_n \gamma_0)
\]
\[
= \left( S_n \prod_{j=1}^n \left\{ (2\pi)^{-3/2} \int_{\mathbb{R}^3} f_j(t, x)U_{t, -x}B_0U_{t, -x}dx \right\} \right) \omega,
\]

Vol. 40, n° 3-1984
where
\[ f_j(t, x) = (2\pi)^{-3/2} \int_{\mathbb{R}^3} e^{-i\mu(p) + i(x \cdot p)} \hat{x}_j(p) dp, \]
i.e. \( f_j(t, x) \) is the spatial inverse Fourier transform of \( e^{-i\mu(p)} \hat{x}_j(p) \), thus \( f_j(t, x) \) is a so-called smooth solution of the Klein-Gordon equation with « negative » frequencies.

**Proof.** — According to Proposition 4 we have the assignment
\[ e^{-it\mu(p)} \hat{x}_j(p) \gamma_0(p) \mapsto (2\pi)^{-3/2} \int_{\mathbb{R}^3} f_j(t, x) U_x B_0 U_{-x} dx. \]
Furthermore,
\[ e^{-i \sum_{j=1}^n \mu(p_j)} \hat{x}_1 \gamma_0 \otimes \ldots \otimes \hat{x}_n \gamma_0 \]
\[ = \bigotimes_{j=1}^n e^{-i\mu(p_j)} \hat{x}_j(p_j) \gamma_0(p_j) = (e^{-itH_0})_n (\hat{x}_1 \gamma_0 \otimes \ldots \otimes \hat{x}_n \gamma_0) \]
is valid, thus we obtain
\[ (Ke^{-itH_0})_n S_n (\hat{x}_1 \gamma_0 \otimes \ldots \otimes \hat{x}_n \gamma_0) \]
\[ = \left( S_n \prod_{j=1}^n \left\{ (2\pi)^{-3/2} \int_{\mathbb{R}^3} f_j(t, x_j) U_x B_0 U_{-x} dx \right\} \right) \omega. \]
Therefore we finally obtain (17) for \( \{ e^{itH}(Ke^{-itH_0}) \}_n S_n (\hat{x}_1 \gamma_0 \otimes \ldots \otimes \hat{x}_n \gamma_0) \)
because of \( e^{itH} = U_{\{ -t, 0 \}} \) and \( U_{\{ t, 0 \}} \omega = \omega. \)

Proposition 6 gives a link between the pre-wave operator \( e^{itH}Ke^{-itH_0} \)
and the expressions (Haag-Ruelle approximants) used in the Haag-Ruelle scattering theory for Wightman fields. That is, Haag-Ruelle’s theory appears as a special part of the abstract two-space scattering theory (see H. Baumgärtel and M. Wollenberg [4]). The basic concept of the abstract scattering theory is given by the pre-wave operator mentioned above. Note that the identification operator \( K \) in the field-theoretic case is unbounded and not closable but densely defined whereas the identification operators appearing usually (e.g. in the non-relativistic scattering theory) are bounded.

It should be mentioned that the characteristic \( \tau \)-dependence of the special field operators \( A(e^{i(t - \mu(p))} f(p)) \) occurring in the Haag-Ruelle approximants (see for example K. Hepp [5, p. 96]) suggests the introduction of a pre-wave operator, i.e. of an operator \( K \). In fact, one obtains, in an intimate connection with Proposition 6, that
\[ (e^{itH}Ke^{-itH_0})_n \left\{ S_n (h(p) \hat{x}_1(p) \otimes \ldots \otimes h(p) \hat{x}_n(p)) \right\} \]
\[ = \left\{ S_n \prod_{\rho=1}^n A(e^{i(t - \mu(p))} h(p) \hat{x}_\rho(p)) \right\} \omega. \]

Annales de l’Institut Henri Poincaré - Physique théorique
is valid (see for example [2, Lemma 6]). Proposition 6 shows explicitly that the pre-wave operator, i.e. also $K$, depends on a single field operator $A(h(p)\gamma_0(p))$ only.

Under the assumptions listed at the beginning of paragraph 1 the strong limits for $t \to \pm \infty$ of (11) exist and they turn out to be isometric (Haag-Ruelle). Because $K$ is uniquely determined by $B_0$ the question arises, what properties of $B_0$ are decisive for the existence and isometry of the wave operators. Some authors (for example A. S. Schwartz [3]) have shown that the (sufficient) assumptions of Haag-Ruelle can be weakened. It would be nice to have some insights what properties of $K$ resp. $B_0$ are necessary for the existence and isometry of the wave operators in order to attack the corresponding inverse problem.

REFERENCES


(Manuscrit reçu le 18 Avril 1983)
Version révisée reçue le 20 Août 1983