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A. V. GOPALAKRISHNA

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Vorticity and Electric Current Behind a Shock Wave in Relativistic Magnetohydrodynamics

by

A. V. GOPALAKRISHNA
Department of Applied Mathematics, Indian Institute of Science, Bangalore 560012, India

ABSTRACT. — By using singular surface theory, it has been shown that in order to find the vorticity jump across a special relativistic magnetohydrodynamic shock of arbitrary shape propagating in a uniform, perfect, infinitely conducting fluid, it is necessary to use an equation of state behind the shock together with the equation of continuity. A general relationship between the jumps in vorticity and electric current vectors, and, in particular the one between the normal components of these have been obtained.

RÉSUMÉ. — En utilisant la théorie des surfaces singulières, on montre que, pour trouver la discontinuité de vorticité, en magnétohydrodynamique relativiste, à la traversée d'un choc de forme arbitraire se propageant dans un fluide parfait uniforme de conductivité infinie, il est nécessaire d'utiliser une équation d'état en arrière du choc en même temps que l'équation de continuité. On obtient une relation générale entre les discontinuités de la vorticité et du vecteur courant électrique, et en particulier entre leurs composantes normales.

1. INTRODUCTION

The study of shock wave propagation in relativistic hydrodynamics and magnetohydrodynamics indicates applications in several contexts of astrophysics [*1*]-[*5*]. Singular surface theory and ray theory provide useful and powerful tools in the study of propagating shocks and weak discontinuities [6]-[12].

Mason [13] studied the behaviour of vorticity in a perfect, relativistic fluid undergoing gravitational collapse. In classical magnetohydrodynamics Kanwal [14] has shown that it is necessary to use energy equation to find the vorticity jump across a shock, and that electric current is also generated behind the shock. In this paper, it is shown that it is necessary to use an equation of state together with the equation of mass conservation to find the jumps in vorticity and electric current across a special relativistic magnetohydrodynamic shock propagating into a constant state ahead.

2. SHOCK CONDITIONS

A shock wave, considered as a propagating, timelike, singular hypersurface Σ across which at least some of the field variables describing the fluid motion are discontinuous, may be represented parametrically by $x^{A} = x^{A}(u^{\alpha})$, where u^{α} (Greek indices assume the values 0, 1, 2) are coordinates on Σ and x^{A} (Latin capital indices range over 0, 1, 2, 3) are the coordinates referred to the special relativity metric $h_{AB} = \mathrm{diag}\,(c^{2}, -1, -1, -1)$. N^{A}, $a_{\alpha\beta}$ and $b_{\alpha\beta}$ denote, respectively, the unit space-like normal vector ($N_{A}N^{A} = -1$), components of the first and second fundamental tensors, of Σ. We note the following formulae [15].

$$x^{A}{}_{,\alpha\beta} = b_{\alpha\beta}N^{A}, \qquad N^{A}{}_{,\alpha} = b^{\beta}_{\alpha}x^{A}{}_{,\beta},$$
$$a^{\alpha\beta}x^{A}{}_{,\alpha}x^{B}{}_{,\beta} = h^{AB} + N^{A}N^{B}. \tag{2.1}$$

In (2.1) and in what follows comma when followed by a Greek index denotes covariant derivative with respect to $a_{\alpha\beta}$ and when followed by a Latin index denotes partial differentiation in x. The vectors $x^{A}{}_{,\alpha}$ are tangent to Σ. Usual summation convention has been followed throughout.

We further note the compatibility conditions, derived by using Hadamard's Lemma, which must be satisfied across Σ by the partial derivatives of the field variables [10]. For the first order partial derivatives, these are

$$[F_{,A}] = -\lambda_{N}N_{A} + a^{\alpha\beta}[F]_{,\alpha}x_{A,\beta}, \tag{2.2}$$

where $[F] = F_{2} - F_{1}$, the subscript 1(2) on F denoting the value of F just ahead (behind) Σ and $\lambda_{N} = [F_{,A}]N^{A}$ denotes the jump in the normal derivative. *The suffix* N *is not a tensor index; it is reserved to denote contraction with* N_{A}, *of a vector under consideration.*

The fluid energy-momentum tensor T^{AB} is given by

$$T^{AB} = \tau V^{A}V^{B} - \frac{1}{c^{2}}ph^{AB} + \frac{\sigma}{c^{2}}\left(\frac{1}{4}G_{CD}G^{CD}h^{AB} - G^{A}_{C}G^{BC}\right), \tag{2.3}$$

where $V^A = \mu U^A$, $\tau = \rho/\mu$, and ρ, p, U^A, σ and μ are, respectively, the proper mass density, pressure, time-like unit velocity 4-vector, the magnetic permeability (assumed constant throughout) and the index of the fluid defined by $\mu = 1 + (e + p/\rho)/c^2$, $e = e(p, \rho)$ being the specific internal energy. The skew-symmetric electromagnetic field tensor G_{AB} is given by

$$G_{AB} = -(1/\mu)\eta_{ABCD}V^C H^D, \tag{2.4}$$

where H^D is the space-like magnetic field vector.

It follows from an analysis of the shock conditions [8] that the vector $Y^A = (1/\mu)(H_N V^A - V_N H^A)$ and the scalars

$$M = \tau V_N, \qquad \eta = \mu H_N, \qquad K = (1/M^2)Y_A Y^A = H_N^2/M^2 - |H|^2/\rho^2,$$

(where $-|H|^2 = H_A H^A$), are all invariant across the shock.

Denoting $[V^A]$ and $[p]$, respectively, by λ^A and Δ, we write the shock condition on energy-momentum tensor as

$$M\lambda^A - \frac{1}{c^2}\Delta N^A + \frac{\sigma}{c^2}\left(\frac{1}{2}\tilde{J}_B N^A - G^A_{B1}\right)\tilde{J}^B = 0, \tag{2.5}$$

noting that $[G_{AB}] = \tilde{J}_A N_B - \tilde{J}_B N_A$, where the « sheet-current density » vector $\tilde{J}^B$ is defined by [16]

$$\tilde{J}^B = [G^{AB}]N_A. \tag{2.6}$$

By eliminating H_D from (2.4) in favour of Y_D and then by using (2.5) and (2.6) one obtains

$$\tilde{J}^B = -([\alpha]/\alpha_2)G_1^{AB}N_A, \tag{2.7}$$

where $\alpha = 1/\tau - \sigma K/c^2$. We assume that α_1 is nonzero and is distinct from α_2. Consequently $\alpha_2 \neq 0$. It then follows from (2.6) and (2.7) that

$$\tilde{J}^B\tilde{J}_B = -[\alpha]^2 k_1^2/\alpha_2^2, \qquad [\alpha G^{AB}]N_A = 0, \tag{2.8}$$

where $k^2 = |H|^2 - M^2 K$ is strictly positive for nonzero magnetic field and hence $\tilde{J}^B$ is space-like. Further $\tilde{J}_B$ is orthogonal to Y_B and V_B (on either side of the shock). From the definitions of M and α we have

$$\lambda_N = M[1/\tau] = M[\alpha]. \tag{2.9}$$

By contracting (2.5) with N_A and then by using (2.8) and (2.9) one obtains

$$\frac{1}{c^2}\Delta = -M^2[\alpha] + \frac{\sigma[\alpha^2]k_1^2}{2c^2\alpha_2^2}. \tag{2.10}$$

The Hugoniot equation [eq. (53.4) in [8]] written as

$$\{ \mu(\mu - \sigma K\rho/c^2) \}_2^2 + (1/c\iota)_2^2 \{ - (\sigma k^2(\alpha + 1/\tau) + (c\mu\iota)^2)_1 \mu_2^2 + (\sigma\eta^2/c^2M^2)(2\rho\mu c^2 - \sigma K\rho^2)_2 \} = 0, \quad (2.11)$$

where $\iota^2 = 1 + M^2/\rho^2$, determines $[\mu]$, and, therefore, Δ, $\tilde{J}^B$ and λ^A, respectively, from (2.10), (2.7) and (2.5), in terms of the state ahead and strength, defined by $[\rho]/\rho_1$, of the shock.

It may be noted that eq. (2.11) also follows from $\mu^2 = V_A V^A$, by taking jumps and then using (2.5)-(2.10).

3. JUMPS IN VORTICITY AND ELECTRIC CURRENT VECTORS

Throughout this section, we assume a *constant state ahead of the shock.* We write the vorticity vector W^A [17] as

$$W^A = (1/\mu)\eta^{ABCD}V_B(V_C/\mu)_{,D}. \quad (3.1)$$

By taking jumps in (3.1) we obtain

$$[W^A] = (1/\mu_2^2)\eta^{ABCD}V_{B2} \{ (\bar{\lambda}^\beta + a^{\nu\beta}\lambda_{N,\nu} - \lambda^\nu b_\nu^\beta)x_{C,\beta}N_D + a^{\nu\beta}\lambda^\gamma{}_{,\nu}x_{C,\gamma}x_{D,\beta} \}, \quad (3.2)$$

where $\bar{\lambda}^A$ denotes the discontinuity

$$\bar{\lambda}^A = - [V^A{}_{,B}]N^B = - \bar{\lambda}_N N^A + \bar{\lambda}^\beta x^A{}_{,\beta}.$$

By applying (2.2) to the equations of motion

$$T^B_{A,B} \equiv \tau V^B V_{A,B} - \frac{1}{c^2} p_{,B} h^B_A - \frac{\sigma}{c^2} G_{AB} J^B = 0, \quad (3.3)$$

where J^B is the electric current vector defined by

$$J^B = G^{AB}{}_{,A}, \quad (3.4)$$

we obtain

$$M\bar{\lambda}_A + \tau_2 \lambda_{A,\beta} V_2^\beta - \frac{1}{c^2}(\bar{\Delta} N_A + a^{\nu\beta}\Delta_{,\nu}x_{A,\beta}) - \frac{\sigma}{c^2} G_{AB2}[J^B] = 0, \quad (3.5)$$

where $\bar{\Delta}$ denotes the discontinuity $\bar{\Delta} = - [p_{,B}]N^B$. By eliminating $\bar{\lambda}^\beta$ between (3.2) and (3.5) and by using (2.1) we obtain the following equation connecting the jumps in vorticity and electric current vectors:

$$[W^A] = (1/\mu_2^2)\eta^{ABCD}V_{B2} \{ (- V_{N2}\lambda^\beta{}_{,\nu}V_2^\nu - (1/\tau_1)b_\nu^\beta[\tau V^\nu] + (1/Mc^2)(\Delta_{,\nu} + Mc^2\lambda_{N,\nu})a^{\nu\beta})x_{C,\beta} + (\sigma/Mc^2)G_{CF2}[J^F] \} N_D - [W_N]N^A, \quad (3.6)$$

where

$$[W_N] = (1/\mu_2^2)\eta^{ABCD}N_A V_{B2} a^{\nu\beta}\lambda^{\gamma}{}_{,\nu} x_{C,\gamma} x_{D,\beta}\,. \qquad (3.7)$$

From (2.4) and (3.4) it follows that

$$[J^A] = -(1/\mu_2)G_2^{BA}[\mu_{,B}] + (1/\mu_2)\eta^{ABCD}\{N_B(\bar{\lambda}_C H_{D2} + V_{C2}\bar{\zeta}_D) + a^{\nu\beta}x_{B,\beta}(V_{C2}H_{D2})_{,\nu}\}, \qquad (3.8)$$

where $\bar{\zeta}_D$ denotes the jump $\bar{\zeta}_D = -[H_{D,B}]N^B$. By taking jumps in Maxwell's equations in the form

$$\left(\frac{V^AH^B - V^BH^A}{\mu}\right)_{,B} = 0\,, \qquad (3.9)$$

we obtain

$$\bar{\lambda}^A H_{N2} + \lambda^A{}_{,\nu}H_2^{\nu} - H_2^A(\bar{\lambda}_N + a^{\nu\beta}\lambda^B{}_{,\nu}x_{B,\beta}) - (V_{N2}\bar{\zeta}^A + \zeta^A{}_{,\nu}V_2^{\nu}) - (1/\mu_2)V_2^{(A}H_2{}^{B)}[\mu_{,B}] = -V_2^A(\bar{\zeta}_N + a^{\nu\beta}\zeta^B{}_{,\nu}x_{B,\beta})\,, \qquad (3.10)$$

where $\zeta^A = [H^A]$ and the brackets around the indices indicate anti-symmetry. By taking jumps in the equations obtained by contracting (3.9) with V_A, we obtain

$$-\mu_2^2(\bar{\zeta}_N + a^{\nu\beta}\zeta^B{}_{,\nu}x_{B,\beta}) = H_2^B(\bar{\lambda}_B V_{N2} + \lambda_{B,\nu}V_2^{\nu})\,. \qquad (3.11)$$

Equations (3.10) and (3.11) give

$$V_{N2}\bar{\zeta}^A = H_{N2}\bar{\lambda}^A + \lambda^A{}_{,\nu}H_2^{\nu} - (V^AH^B/\mu^2)_2(\bar{\lambda}_B V_{N2} + \lambda_{B,\nu}V_2^{\nu}) - (\bar{\lambda}_N + a^{\nu\beta}\lambda^B{}_{,\nu}x_{B,\beta})H_2^A - \zeta^A{}_{,\nu}V_2^{\nu} - (1/\mu_2)V_2^{(A}H_2^{B)}[\mu_{,B}]\,. \qquad (3.12)$$

Now eliminate $\bar{\lambda}_C$ and $\bar{\zeta}_D$ between (3.5), (3.12) and (3.8) (using (2.2) for $[\mu_{,B}]$) and in the resulting equation eliminate the magnetic field in favour of the invariant vector Y_D to obtain

$$[J^A] = a^{\nu\beta}G_{2,\nu}^{BA}(x_{B,\beta} - (\tau_2/M)N_B V_{\beta 2}) - (\tau_2/M)G_2^{BA}N_B(\bar{\lambda}_N + a^{\nu\beta}\lambda^C{}_{,\nu}x_{C,\beta}) + (\tau_2/M)^2\eta^{ABCD}N_B\{\lambda_{C,\nu}Y^{\nu}V_{D2} - (1/c^2\tau_2)a^{\nu\beta}\Delta_{,\nu}x_{C,\beta}Y_D - (\sigma/c^2\tau_2)G_{CF2}[J^F]Y_D\}\,. \qquad (3.13)$$

Now we show that the jump in the normal component of the electric current vector is the surface divergence of the sheet-current density vector.

By contracting (3.13) with N_A we get

$$\begin{aligned}[J_N] &= a^{\nu\beta}G_{2,\nu}^{BA}x_{B,\beta}N_A \\ &= a^{\nu\beta}(G_2^{BA}N_A)_{,\nu}x_{B,\beta} \\ &= a^{\nu\beta}(G_1^{BA}N_{A,\nu} - \tilde{J}^B{}_{,\nu})x_{B,\beta} \\ &= G_1^{BA}b^{\nu\beta}x_{A,\nu}x_{\beta,\beta} - \tilde{J}^{\beta}{}_{,\beta} + b_\beta^\beta\tilde{J}^B N_B\,.\end{aligned}$$

It is clear that the first and the last terms in the right hand side vanish and hence we get

$$- [J_N] = \tilde{J}^{\beta}{}_{,\beta}, \tag{3.14}$$

proving the statement made above.

By using the results obtained so far and properties of the permutation tensor one obtains

$$\eta^{ABCD} N_B G_{CF2} Y_D [J^F] = - M^2 K [J^A] + (\tau_2/M) \tilde{J}^{\beta}{}_{,\beta} (\eta Y^A - M^2 K V_2^A) + [J^B] Y_B Y^A, \tag{3.15}$$

with $[J^B]Y_B = (\tau_2/M)\lambda_A \tilde{J}^A{}_{,\beta} Y^\beta$. By using (3.15) and the definition of the variable α in (3.13) one gets

$$\begin{aligned} M^3\tau_2\alpha_2[J^A] = {} & M^2 a^{\nu\beta} G_2^{BA}{}_{,\nu}(M x_{B,\beta} - \tau_2 V_{\beta 2} N_B) \\ & - M^2 \tau_2 G_2^{BA} N_B (\bar{\lambda}_N + a^{\nu\beta} \lambda^C{}_{,\nu} x_{C,\beta}) \\ & + M\tau_2 \eta^{ABCD} N_B \{ \tau_2 \lambda_{C,\beta} Y^\beta V_{D2} - (1/c^2) a^{\nu\beta} \Delta_{,\nu} x_{C,\beta} Y_D \} \\ & - (\sigma\tau_2^2/c^2) \{ \tilde{J}^\beta{}_{,\beta}(\eta Y^A - M^2 K V_2^A) + \lambda_B \tilde{J}^B{}_{,\beta} Y^\beta Y^A \}. \end{aligned} \tag{3.16}$$

It is clear from (3.6) and (3.16) that in order to find $[W^A]$ and $[J^A]$ in terms of the strength, curvature and state ahead of the shock, one has to determine $\bar{\lambda}_N$ and the tangential derivatives of the various jumps. To find these latter quantities, as we have no more equations at our disposal, we now assume an equation of state behind the shock and use it in conjunction with the equation of continuity as follows. It may be noted that the use of an equation of state is not necessary to find the vorticity jump in the absence of magnetic field [12].

By taking jumps in the equation of continuity we obtain

$$[\rho_{,\beta}] V_2^B - \tau_2 [\mu_{,B}] V_2^B + \rho_2 [V^B{}_{,B}] = 0. \tag{3.17}$$

We now assume that $\mu = \mu(p, \rho)$ is a *given* function of p and ρ behind the shock so that the partial derivatives of μ with respect to p and ρ, denoted respectively by f and g, are known functions. Then we have

$$[\mu_{,B}] V_2^B = f_2 [p_{,B}] V_2^B + g_2 [\rho_{,B}] V_2^B. \tag{3.18}$$

By eliminating $[\rho_{,B}] V_2^B$ between (3.17) and (3.18) we get

$$(1 - \tau g)_2 [\mu_{,B}] V_2^B = f_2 [p_{,B}] V_2^B - (\rho g)_2 [V^B{}_{,B}]. \tag{3.19}$$

By taking jumps in the energy equation (obtained on contraction of (3.3) with V^A) we obtain

$$\rho_2 [\mu_{,B}] V_2^B = (1/c^2) [p_{,B}] V_2^B = (1/c^2) [\bar{\Delta} V_{N2} + \Delta_{,\beta} V_2^\beta], \tag{3.20}$$

where the last line is obtained by applying (2.2) to $[p_{,B}]$. From (3.19) and (3.20) we get

$$(\rho g)_2 (\bar{\lambda}_N + a^{\nu\beta} \lambda^A{}_{,\nu} x_{A,\beta}) = h_2 (\bar{\Delta} V_{N2} + \Delta_{,\beta} V_2^\beta), \tag{3.21}$$

where $h = f - \{(1 - \tau g)/c^2\rho\}$. Now eliminate $\overline{\Delta}$ between (3.21) and (3.5) to get

$$(\rho g\tau + c^2\mathrm{M}^2h)_2\overline{\lambda}_\mathrm{N} = (-c^2\mathrm{M}\mathrm{N}^\mathrm{A}\lambda_{\mathrm{A},\beta} + \Delta_{,\beta})(h\tau\mathrm{V}^\beta)_2 - (\rho g\tau)_2 a^{\nu\beta}\lambda^\mathrm{A}{}_{,\nu}x_{\mathrm{A},\beta} - (\sigma\mathrm{M}\alpha_1/[\alpha])h_2\tilde{\mathrm{J}}_\mathrm{B}[\mathrm{J}^\mathrm{B}]. \quad (3.22)$$

Equations (3.16) and (3.22) may be considered as a system of two linear equations for $\overline{\lambda}_\mathrm{N}$ and $[\mathrm{J}^\mathrm{A}]$. Thus the solution of this system produces the required $[\mathrm{J}^\mathrm{A}]$ which on substitution in (3.6) determines the vorticity jump.

It is interesting to look at the relation between the normal components of the jumps in vorticity and electric current vectors. It follows from (2.5) and (3.7), after some simplifications, that

$$[\mathrm{W}_\mathrm{N}] = (1/\mathrm{M}\mu_2^2)\eta^{\mathrm{ABCD}}\mathrm{N}_\mathrm{A}(\lambda_\mathrm{B} + \mathrm{V}_{\mathrm{B}1})a^{\nu\beta}\{(\sigma/c^2)(([\alpha]k_1^2/\alpha_2)b_\nu^\gamma c_{\mathrm{C},\gamma} + \mathrm{G}_{\mathrm{CF}1}\tilde{\mathrm{J}}^\mathrm{F}{}_{,\nu} - (\alpha_2/[\alpha])\tilde{\mathrm{J}}_\mathrm{F}\tilde{\mathrm{J}}^\mathrm{F}{}_{,\nu}\mathrm{N}_\mathrm{C}) - \mathrm{M}_{,\nu}\lambda^\gamma x_{\mathrm{C},\gamma}\}x_{\mathrm{D},\beta}. \quad (3.23)$$

In view of the skew-symmetry of the permutation tensor the terms involving the second fundamental tensor and N_C, and the last term when considered with λ_B, in the right hand side of (3.23), vanish. Thus we have

$$[\mathrm{W}_\mathrm{N}] = (1/\mathrm{M}\mu_2^2)\eta^{\mathrm{ABCD}}\mathrm{N}_\mathrm{A}a^{\nu\beta}\{(\sigma/c^2)(\lambda_\mathrm{B} + \mathrm{V}_{\mathrm{B}1})\mathrm{G}_{\mathrm{CF}1}\tilde{\mathrm{J}}^\mathrm{F}{}_{,\nu} - \mathrm{V}_{\mathrm{B}1}\mathrm{M}_{,\nu}\lambda_\mathrm{C}\}x_{\mathrm{D},\beta}, \quad (3.24)$$

where we have written $\lambda^\gamma x_{\mathrm{C},\gamma} = \lambda_\mathrm{C} + \lambda_\mathrm{N}\mathrm{N}_\mathrm{C}$, again in view of the skew-symmetry of the permutation tensor. We now note that

$$\mathrm{M}_{,\nu} = \tau_1 b_\nu^\beta \mathrm{V}_{\beta 1}. \quad (3.25)$$

By carrying out the necessary algebra, we obtain

$$\eta^{\mathrm{ABCD}}\mathrm{N}_\mathrm{A}(\lambda_\mathrm{B} + \mathrm{V}_{\mathrm{B}1})\mathrm{G}_{\mathrm{CF}1}a^{\nu\beta}\tilde{\mathrm{J}}^\mathrm{F}{}_{,\nu}x_{\mathrm{D},\beta} = ((\sigma[\alpha]\,|\,\mathrm{H}\,|_1^2/\mathrm{M}c^2\alpha_2)\mathrm{Y}^\nu - \mu_1\mathrm{H}_1^\nu)b_\nu^\beta\tilde{\mathrm{J}}_\beta - \eta\tilde{\mathrm{J}}^\beta{}_{,\beta}, \quad (3.26)$$

and

$$\eta^{\mathrm{ABCD}}\mathrm{N}_\mathrm{A}\mathrm{V}_{\mathrm{B}1}\lambda_\mathrm{C}x_{\mathrm{D},\beta} = (\sigma\eta/\mathrm{M}c^2)\tilde{\mathrm{J}}_\beta. \quad (3.27)$$

By using (3.25)-(3.27) and (3.14) in (3.24) we obtain

$$[\mathrm{W}_\mathrm{N}] = (\sigma\eta/\mathrm{M}c^2\mu_2^2)[\mathrm{J}_\mathrm{N}] + (\sigma/(\mathrm{M}c\mu_2)^2)\{\rho_1\mu_1 + (\sigma[\alpha]\,|\,\mathrm{H}\,|_1^2/c^2\alpha_2)\}b_\nu^\beta\tilde{\mathrm{J}}_\beta\mathrm{Y}^\nu, \quad (3.28)$$

where the tangential magnetic field occuring in (3.26) is eliminated with the help of the invariant vector Y^A. It is now clear from (3.28) that the discontinuity in vorticity across a shock (in particular a plane shock) gives rise to a discontinuity in the electric current.

Finally, the tangential derivatives of the equations (2.10), (2.11) and the known function $\mu_2 = \mu_2(p_2, \rho_2)$ together determine all the tangential derivatives in (3.6) and (3.16) in terms of the strength $[\rho]/\rho_1$, curvature and state ahead of the shock. Hence $[W^A]$ and $[J^A]$ are known in terms of the latter quantities. For example, in the case of transverse magnetic field and for the choice $\mu = 1 + \beta p/\rho$, $\beta = \gamma/c^2(\gamma - 1)$, γ (assumed constant) being the ratio of specific heats, equations (2.10) and (2.11) give

$$\rho_1^2\alpha_1(M^2c^2\beta + \rho_2^2)[\sqrt{\rho^2 + M^2}] = [\rho]\{\rho_2(\beta V + \rho)_1 + (\beta\rho V)_1\}\sqrt{\rho_2^2 + M^2}, \tag{3.29}$$

where $V = p + \sigma |H|^2/2$ is the total pressure.

It then follows from (3.25) and (3.29) that $\rho_{2,v}$ can be calculated in terms of the curvature, strength and state ahead of the shock. In the absence of the magnetic field, eq. (3.29), omitting the details, reduces under the strong shock approximation to the cubic eqn. for ρ_2/ρ_1 obtained by Guess [*18*].

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