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Section A :
Physique théorique.

Construction of U-gauge Green's functions of gauge theories with spontaneous symmetry breaking and Slavnov identities

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ABSTRACT. — We construct the unitary (U) gauge Green's functions of the U(1) and SU(2) Higgs-Kibble models applying a renormalized point transformation and a non-local gauge changing transformation to a manifestly renormalizable (R gauge) version of the respective theory. It is shown that the cancellation mechanism known as « tree graph unitarity » rendering in tree graph approximation a smooth high energy behaviour of the U gauge Green's functions on mass shell can in a natural way be extended to all orders of perturbation theory. We prove that the conditions imposed by this « generalized tree graph unitarity » on the renormalization programme are equivalent with the requirement of renormalized Slavnov identities for the R gauge Green's functions.

RÉSUMÉ. — Nous construisons dans la jauge unitaire (U) les fonctions de Green des modèles de Higgs-Kibble U(1) et SU(2) en appliquant une transformation ponctuelle renormalisée et une transformation non locale qui change la jauge à une version (jauge R) manifestement renormalisable de ces théories.

Nous montrons que le mécanisme de compensation connu sous le nom « d'unitarité à l'approximation en arbre » qui donne à cette approximation un comportement mou à haute énergie s'étend naturellement aux ordres supérieurs de la théorie de perturbation.

Nous prouvons que les contraintes de renormalisation imposées par cette « unitarité à l'approximation d'arbre généralisée » sont équivalentes à celles imposées par les identités de Slavnov renormalisées pour les fonctions de Green dans la jauge R.

1. INTRODUCTION

There exists up to now a number of elaborate proofs ([*1*]-[*3*]) for the renormalizability of various gauge field theories with a mass generating spontaneous symmetry breaking. It is common to all these approaches that the renormalization is set up in one or another manifestly renormalizable (R-gauge) version of the respective model. The major problem is then to show that a certain amount of gauge invariance is preserved throughout the renormalization procedure, in order to assure the decoupling of the unphysical from the physical degrees of freedom. The unphysical degrees have to be introduced for the sake of manifest renormalizability. One might rephrase this problem in the following way. A classical Lagrange density of the Yang-Mills type, which looks non-renormalizable because of the bad high-energy behavior of the vector boson propagators, is replaced via a point transformation and a subsequent non-local gauge changing transformation by a manifestly renormalizable R-gauge Lagrangian. The troublesome longitudinal parts of the vector field have been thereby eliminated on cost of the introduction of some new unphysical degrees of freedom. Both transformations are unproblematic on tree graph level in the sense that one can easily verify in this approximation on-shell invariance of all amplitudes against the transformations. The very existence of such a mass shell invariant interpolation to a manifestly renormalizable Lagrangian guarantees a smooth high-energy behavior of the scattering amplitudes. This property has been called « tree graph unitarity » [*4*]. The problem of renormalization is found in the proof that the cancellation mechanism responsible for tree graph unitarity survives the renormalization of the higher order quantum corrections.

We want to demonstrate in this paper that the inductive renormalization scheme of Epstein and Glaser [*5*] is the natural frame to display the desired cancellation mechanism, establishing at the same time in explicit form the connection between renormalizability and tree graph unitarity. We believe that, along the lines of the present paper, one can work out a general proof that tree graph unitarity implies renormalizability.

The main tool of our approach will be a simple proof for the invariance of the S-matrix under point transformations of the field variables [*6*]. This will enable us to construct the U-gauge Green's functions by interpolation from the renormalized R-gauge theory. It will turn out that renormalized Slavnov identities for the R-gauge theory are the necessary and sufficient conditions for the interpolated U-gauge theory to be in fact unitary and to have a smooth high-energy mass shell hehaviour.

Our procedure is in a technical sense complementary to the approach of Becchi, Rouet and Stora (BRS) [*3*]. These authors use the requirement of Slavnov identities for the R-gauge Green's functions as a basic constructive principle inferring therefrom gauge invariance and unitarity of the S-matrix, whereas we insist on manifest unitarity from the beginning. Slavnov identities appear in our approach as equivalent to conditions necessary to implement consistently the cancellation mechanism of tree graph unitarity into the quantum corrections.

As our procedure relies heavily on mass shell equivalence, we can treat rigorously only models without massless particles. We select as concrete examples for our discussion the abelian U(1) and the non-abelian SU(2) Higgs-Kibble models [7].

In Section 2 of this paper we recapitulate the rules taken over from [6] which have to be followed in the renormalization of a point-transformed Lagrangian rendering S-matrix invariance. Section 3 is devoted to a discussion of the abelian Higgs-Kibble model. In Section 4 we describe the modifications of arguments needed for the SU(2) model in comparison with the abelian theory.

2. THE INVARIANCE OF THE S-MATRIX UNDER POINT TRANSFORMATIONS

For simplicity we consider in this section a Lagrangian with exclusively scalar particles. The generalization of our procedure to vector particles is straightforward.

Let $\mathscr{L}$ and $\mathscr{L}'$ be two Lagrangians related through a point transformation

$$\begin{aligned} \mathscr{L} &= \mathscr{L}(\phi, \partial_\mu \phi), \\ \mathscr{L}'(\phi) &= \mathscr{L}(\phi + h, \partial_\mu(\phi + h)), \end{aligned} \tag{1}$$

$\phi = \{ \phi_i, 1 \leqslant i \leqslant n \}$ denotes a collection of scalar particles fields with possibly different masses $m_i > 0$ and $h = \{ h_i \}$, the corresponding collection of transformation functions, which are supposed to have a formal power series expansion in the fields ϕ_i such that

$$\begin{gathered} h_i(z)\Big|_{z=0} = \frac{\partial h_i}{\partial z_j}\Big|_{z=0} = 0, \\ 1 \leqslant i, j \leqslant n, \qquad z = (z_1, \ldots, z_n). \end{gathered} \tag{2}$$

The physical equivalence of the two Lagrangians (1) and (1 *a*) in the classical approximation, that is on tree graph level, is a standard result of classical field theory. To start with we re-derive this result in a form adapted to our purposes.

It is convenient to consider an interpolating Lagrangian

$$\mathscr{L}_\lambda = \mathscr{L}(\phi + \lambda h, \partial_\mu(\phi + \lambda h)), \qquad 0 \leqslant \lambda \leqslant 1$$

and to construct the derivative of the $\mathscr{L}_\lambda$ Green's functions with respect to the parameter λ.

Let
$$\int d^4x_1 \dots d^4x_n \{ \mathrm{T}(: g_1(x_1) : \dots : g_n(x_n) :) \}^t$$
denote the set of tree graphs corresponding to the time-ordered product of the normal-ordered Fock space operators $: g_1 : \dots : g_n :$. The set of all tree graphs generated by the interaction part $\bar{\mathscr{L}}_\lambda = \mathscr{L}_\lambda - 1/2\partial_\mu\phi\partial_\mu\phi + 1/2m_i^2\phi_i^2$ (1) of $\mathscr{L}_\lambda$ is in this notation

$$\mathrm{T}(\lambda) = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int dx_1 \dots dx_n \{ \mathrm{T}(: \bar{\mathscr{L}}_\lambda(x_1) : \dots : \bar{\mathscr{L}}_\lambda(x_n) :) \}^t ,$$
$$\frac{d\mathrm{T}(\lambda)}{d\lambda} = \sum_{n=0}^{\infty} \frac{i^{n+1}}{n!} \int dx_1 \dots dx_n dx_{n+1} \left\{ \mathrm{T}\left(: \frac{d\bar{\mathscr{L}}_\lambda(x_1)}{d\lambda} : : \bar{\mathscr{L}}_\lambda(x_2) : \dots : \bar{\mathscr{L}}_\lambda(x_{n+1}) : \right) \right\}^t \tag{3}$$

$d\bar{\mathscr{L}}/d\lambda$ can be manipulated as follows:

$$\frac{d\bar{\mathscr{L}}}{d\lambda} = \partial_\mu \left(\frac{\partial \bar{\mathscr{L}}}{\delta(\partial_\mu(\phi_i + \lambda h_i))} h_i \right) - h_i(\square + m_i^2)\phi_i + \mathrm{R}_{\lambda;i}\mathrm{A}_{ik}h_k ,$$
$$\mathrm{R}_{\lambda;i} = \bar{\mathscr{L}}_{\lambda;\phi_i} + \lambda \frac{\partial h_k}{\partial \phi_i}(\square + m_k^2)\phi_k , \tag{4}$$
$$\bar{\mathscr{L}}_{\lambda;\phi_i} = \frac{\delta \bar{\mathscr{L}}_\lambda}{\delta \phi_i} - \partial_\mu \left(\frac{\delta \bar{\mathscr{L}}}{\delta(\partial_\mu \phi_i)} \right), \qquad \mathrm{A}_{ik} = \left(\delta_{ik} + \lambda \frac{\partial h_i}{\partial \phi_k} \right)^{-1} .$$

We insert (4) into Eq. (3). The terms with total derivatives in front drop out. Of special interest is the contribution of $h_i(\square + m_i^2)\phi_i$ to (3):

$$i^{n+1} \int dx_1 \dots dx_n \{ \mathrm{T}(: \bar{\mathscr{L}}_\lambda(x_1) : \dots : (-h_i)(\square + m_i^2)\phi_i(x_j) : \dots : \bar{\mathscr{L}}_\lambda(x_n) : \}^t$$
$$= i^{n+1} \int dx_1 \dots dx_{n+1} \{ \mathrm{T}(: \underline{h_i(\square + m_i^2)\phi_i} : \dots) \}^t \tag{5}$$
$$- \sum_{l=2}^{n+1} i^n \int dx_2 \dots dx_{n+1} \{ \mathrm{T}(: \bar{\mathscr{L}}_\lambda(x_2) : \dots : \bar{\mathscr{L}}_{\lambda;\phi_i} h_i(x_l) : \dots : \bar{\mathscr{L}}_\lambda(x_{n+1}) :) \}^t$$

The first term stands for the graphs not contributing to the mass shell, where $(\square + m^2)$ acts on an external line denoted by an underline. The second term represents all possibilities in which $(\square + m^2)$ acts on an internal propagator line, thereby contracting it to a δ-function.

Writing for $\bar{\mathscr{L}}_{\lambda;l}h_l$
$$\bar{\mathscr{L}}_{\lambda;l}h_l = - h_l \lambda \frac{\partial h_k}{\partial \phi_l}(\square + m_k^2)\phi_k + \mathrm{R}_{\lambda;l}h_l$$

(1) Repeated indices are summed over.

we can perform further contractions:

$$i^n \int dx_1 \ldots dx_{n+1}$$
$$\left\{ \mathrm{T}\left(: \bar{\mathscr{L}}_\lambda(x_1) : \ldots : h_l \lambda \frac{\partial h_k}{\partial \phi_l} (\Box + m_k^2)\phi_k(x_l) : \ldots : \bar{\mathscr{L}}_\lambda(x_n) : \right) \right\}^t$$
$$= i^n \int dx_2 \ldots dx_{n+1}$$
$$\left\{ \mathrm{T} : h_l \lambda \frac{\partial h_k}{\partial \phi_l} \underline{(\Box + m_k^2)\phi_k(x_l)} : \ldots \right\}^t + i^{n-1} \sum_{p=1}^{n+1} \int dx_2 \ldots dx_{n+1}$$
$$\left\{ \mathrm{T}\left(: \bar{\mathscr{L}}_\lambda(x_2) : \ldots : h_l \lambda \frac{\partial h_k}{\partial \phi_l} \bar{\mathscr{L}}_{\lambda;\phi_k}(x_p) : \ldots : \bar{\mathscr{L}}_\lambda(x_{n+1}) : \right) \right\}^t . \quad (6)$$

Equations (5) and (6) are the first steps in a rearrangement of tree graphs. Continuing the procedure, it is easy to see that the sum of the contracted graphs, not trivially vanishing on the mass shell, add up to geometric series

$$- \int dx_1 \ldots dx_n \left\{ \mathrm{T}(: \bar{\mathscr{L}}_\lambda(x_1) : \ldots : \mathrm{R}_{\lambda;k} \mathrm{A}_{ki} h_i(x_l) : \ldots : \bar{\mathscr{L}}_\lambda(x_n) : \right\}^t \quad (7)$$

and thereby cancel with the graphs of (3) on which no contractions have been performed.

The building blocks of the preceding argument were first a successive use of partial integrations substituting, for example, in (4) $\partial_\mu h \partial_\mu \phi$ by $(- h\Box\phi)$—the contribution of $\partial_\mu(h\partial_\mu\phi)$ drops out integrating over the vertex— and second, an extensive use of the equation of motion for the free field Feynman propagator

$$(\Box_x + m^2)\Delta_f(x - y, m) \sim \delta^{(4)}(x - y) .$$

Graphs with external lines contracted to δ-functions do not contribute on mass shell, because they have no one particle pole in the momentum variable conjugate to the end point of the contracted external line.

To illustrate how these elements of proof for the mass shell equivalence of the two Lagrangians $\mathscr{L}_{\lambda=0}$ and $\mathscr{L}_{\lambda=1}$ can be pushed to higher orders of perturbation theory, we start with an inspection of graphs with two vertices corresponding to the time-ordered operator product ([2])

$$\mathrm{T}^{(2)}(x, y) \equiv \mathrm{T}\left(: \frac{d\bar{\mathscr{L}}(x)}{d\lambda} : : \bar{\mathscr{L}}_\lambda(y) : \right)$$

([2]) To simplify notations, we use in the following, always as shorthand notation, operator expressions for the corresponding set of (non-vacuum) graphs. The triple points in Eq. (8) shall denote the specific renormalization prescriptions we impose later on.

$T^{(2)}$ can be represented as the sum of a (well-defined) Wightman two-point function

$$R^{(2)\prime} = :\frac{d\mathscr{L}_\lambda}{d\lambda}(x):\ :\mathscr{L}_\lambda(y):$$

and a retarded commutator $R^{(2)}$, which is formally given by

$$R^{(2)} = \theta(x^0 - y^0)\left[:\frac{d\mathscr{L}(x)}{d\lambda}:,\ :\mathscr{L}_\lambda(y):\right] \tag{9}$$

$$T^{(2)} = R^{(2)} - R^{(2)\prime} \tag{10}$$

The renormalization of $T^{(2)}$ is synonymous with the problem to give the multiplication of the commutator in (9) with a step function—what Epstein and Glaser call cutting—a precise meaning. For the details of the cutting construction we refer to [5]. It is sufficient for our purposes to note that the following manipulations are compatible with it:

1) Decomposing the commutator $[d\mathscr{L}_\lambda/d_\lambda, \mathscr{L}_\lambda]$ into a sum [cf. Eq. (4)]

$$\left[:\frac{d\mathscr{L}_\lambda}{d\lambda}:,\ :\mathscr{L}_\lambda:\right] = \left[:\partial_\mu\left(\frac{\delta\mathscr{L}_\lambda}{\delta(\partial_\mu(\phi_i + \lambda h_i))}h_i\right):,\ :\mathscr{L}_\lambda:\right]$$
$$+ [:-h_i(\square + m_i^2)\phi_i:,\ \mathscr{L}_\lambda] + [:R_{\lambda;i}A_{ik}h_k:,\ \mathscr{L}_\lambda], \tag{11}$$

one might perform the cutting for each commutator separately.

2) The retarded commutator corresponding to the first term on the right-hand side of (11) can be constructed so that

$$R\left(:\partial_\mu\left(\frac{\delta\mathscr{L}_\lambda}{\delta(\partial_\mu(\phi_i + \lambda h_i))}\,.\,h_i(x)\right):,\ :\mathscr{L}_\lambda(y):\right) = \partial_\mu R(\,,\,) \tag{12}$$

holds, assuring therewith that this term does not contribute in the adiabatic limit.

3) Concerning the second term in (11), we consider graphwise alternatives: tree graphs with an operator $(\square + m_i^2)$ applied to an external respective internal line (fig. 1 *a*, 1 *b*) and the analogous distinction for loop graphs (fig. 1 *c*, 1 *d*) ([3]).

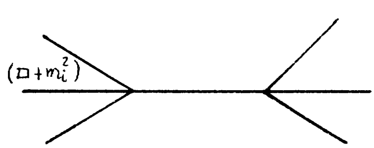

FIG. 1 *a*

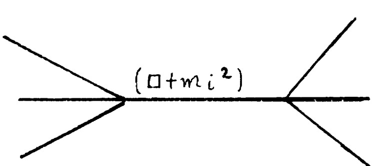

FIG. 1 *b*

([3]) The internal lines of these graphs are supposed to be free field Wightman functions $\Delta_\pm(x - y) = \int \delta(p^2 - m_i^2)\theta(\pm p_0)e^{ip(x-y)}d^4p$ and the external lines Feynman propagators $\Delta_f(x, m_i) \sim (e^{ipx}d^4p/p^2 - m_i^2 + i\varepsilon)$.

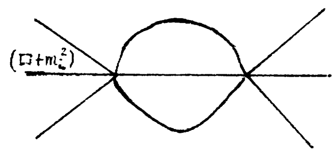

FIG. 1 *c*.

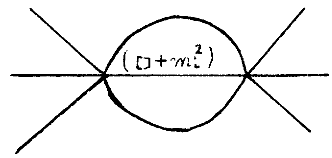

FIG. 1 *d*.

The commutator diagrams of fig. 1 *b* and 1 *d* drop out because the $\Delta_\pm$ functions restrict the momentum flow from one vertex to the other to the mass shell, i. e. $(\Box_x + m_i^2)\Delta_\pm(x, m_i) \equiv 0$. Passing over to the retarded commutator, we pick up a contact term

$$\delta(x-y) : (-h_i)\bar{\mathscr{L}}_{\lambda;\phi_i}(y) :$$

which one might visualize to arise from the contraction of the internal line of the tree graphs in fig. 1 *b*. Terms which would formally originate from contractions on graphs of fig. 1 d, building up diagrams of a type displayed in fig. 2, are suppressed.

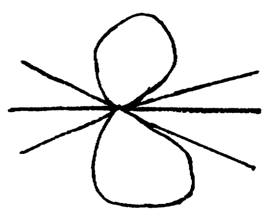

FIG. 2.

To arrange the contact terms in this way we exploited the margin of the « axioms of renormalization » [*8*], which allows us to add local counterterms.

Performing the cutting of the commutator (11) according to the recipes in (1) to (3) and decomposing $: d\mathscr{L}_\lambda/d\lambda :$ in the Wightman two-point function $R^{(2)}$ as before in Eq. (11), we arrive at the following relation for $T^{(2)}$

$$\begin{aligned} T^{(2)} = i^2 T\left(: \frac{d\bar{\mathscr{L}}_\lambda}{d\lambda}(x) : : \bar{\mathscr{L}}_\lambda(y) : \right) \\ = i^2 \left\{ \partial_{x\mu} T\left(: \frac{\delta\bar{\mathscr{L}}_\lambda}{\delta(\partial_\mu(\phi_i + \lambda h_i))} h_i(x) : : \bar{\mathscr{L}}_\lambda(y) : \right) \right. \\ + T(: (-h_i)\underline{(\Box + m_i^2)\phi_i(x)} : : \bar{\mathscr{L}}_\lambda(y) : \\ \left. + T(: R_{\lambda;i}\overline{A_{ik}h_k(x) : : \bar{\mathscr{L}}_\lambda(y)} : \right\} + i\delta^{(4)}(x-y) : (-h_i)\bar{\mathscr{L}}_{\lambda;\phi_i}(y) : \quad (13) \end{aligned}$$

where the underline of $(\Box + m_i^2)\phi_i$ shall denote here and in the following those terms of the Wick expansion in which $(\Box + m_i^2)\phi_i$ is combined with an external field.

Equation (13) is the first of the following series of identities to be proven by induction:

$$
\begin{aligned}
\mathrm{T}^{(n+1)} &\equiv i^{n+1}\mathrm{T}\left(:\frac{d\bar{\mathscr{L}}_\lambda(x_1)}{d\lambda}::\bar{\mathscr{L}}_\lambda(x_2): \ldots :\bar{\mathscr{L}}_\lambda(x_{n+1}): \right) \\
&= i^{n+1}\left\{ \partial_{x_1\mu}\mathrm{T}\left(:\frac{\delta\bar{\mathscr{L}}_\lambda}{\delta(\partial_\mu(\phi_i+\lambda h_i))} h_i(x_1): \ldots \right) + \mathrm{T}(:(-h_i)\underline{(\square+m_i^2)\phi_i(x_1)}: \ldots) \right\} \\
&+ i^n \sum_{l=2}^{n+1} \delta(x_1-x_l)\mathrm{T}(:\bar{\mathscr{L}}_\lambda(x_2): \ldots :(-h_i)\bar{\mathscr{L}}_{\lambda;\phi_i}(x_l): \ldots :\bar{\mathscr{L}}_\lambda(x_{n+1}):), \qquad (14)
\end{aligned}
$$

$$1 \leqslant n \leqslant \infty$$

For the single terms in the sum $i^n \sum_{l=2}^{n+1}(\ldots)$ on the right-hand side of (14), one should be able to write

$$
\begin{aligned}
&i^n\mathrm{T}(\ldots :(-h_i)\bar{\mathscr{L}}_{\lambda;\phi_i}(x_l): \ldots) \\
&= i^n\mathrm{T}\left(\ldots :(-h_i)\left(\mathrm{R}_{\lambda;l}\mathrm{A}_{li} - \lambda\frac{\partial h_k}{\partial\phi_i}(\square+m_k^2)\phi_k \right): \ldots \right) \\
&= i^n\mathrm{T}\left(\ldots :(-h_i)\mathrm{R}_{\lambda;l}\mathrm{A}_{li} - h_i\lambda\underline{\frac{\partial h_k}{\partial\phi_i}(\square+m_k^2)\phi_k(x_l)} \right) \\
&+ i^{n-1}\sum_{\substack{j=2 \\ j\neq l}}^{n+1} \delta^{(4)}(x_j-x_l)\mathrm{T}\left(:\bar{\mathscr{L}}_\lambda(x_2): \ldots :h_i\lambda\frac{\partial h_k}{\partial\phi_i}\bar{\mathscr{L}}_{\lambda;\phi_k}(x_j):\bar{\mathscr{L}}_\lambda(x_{n+1}): \right). \qquad (15)
\end{aligned}
$$

We do not write down the further identities obtained by iteration from (14) and (15), singling out at each step a term with a free field null operator $(\square + m_i^2)\phi_i$ generating therewith in the n^{th} iteration step a vertex $:h_k(\mathrm{B}^n)_{kl}\bar{\mathscr{L}}_{\lambda;\phi l}:$, where $(\mathrm{B})_{kl} = [-\lambda(\partial h_l/\partial\phi_k)]$.

Relations (14) and (15) and the relations derived from them by iteration are, in their formal appearance, exactly the same as those we used before to prove on tree graph level S-matrix invariance of the point transformation. The combinatorics of the on-shell cancellations therefore work out in the general case, including all sorts of loop graphs entirely analogous to the tree graph cancellations. The graphical imagination one might develop of Eqs. (14), (15), etc. is the following: starting at the vertex $d\bar{\mathscr{L}}_\lambda/d\lambda(x_1)$, one generates new vertices, contracting all propagators with an operator $(\square + m_i^2)$ acting on, which connect x_1 with either an external field or another vertex, say, x_j, such that no other propagator connects x_1 and x_j —we call this situation a « tree like » $(t1)$ connection of x_1 and x_j. Going on with contractions of $(t1)$ connections at x_j and at all other vertices in which x_1 has already been contracted, and tracing finally the procedure through all graphs of a given order, one ends up with the right-hand side of Eqs. (14) and (15) and their iterated daughters. Graphs in which an operator $(\square + m_i^2)$ acts on a propagator constituting a non-$t1$ connection (i. e. there is at least one other propagator connecting the same two vertices) are suppressed.

For an inductive proof of relations (14) and (15) and their daughters, we assume that the time-ordered functions

$$\mathrm{T}^{(j)} \equiv \mathrm{T}\left(: \frac{d\mathscr{L}_\lambda}{d\lambda}(x_1) : : \mathscr{L}_\lambda(x_2) : \ldots : \mathscr{L}_\lambda(x_j) : \right)$$

with less than $(n + 1)$ points have already been defined satisfying these relations. In order to construct $\mathrm{T}^{(n+1)}$ along Epstein and Glaser's lines, we consider first

$$\mathrm{D}^{(n+1)}(x_1 ; x_2 \ldots x_{n+1}) = \sum_{\substack{\mathrm{J} \cup \mathrm{J}' = \{x_2, \ldots, x_{n+1}\} \\ \mathrm{J} \cap \mathrm{J}' = \varnothing \\ \mathrm{J} \neq \varnothing}} [\mathrm{T}(x_1, \mathrm{J}'), \bar{\mathrm{T}}(\mathrm{J})](-1)^{|\mathrm{J}|} \quad (16)$$

where

$$\mathrm{T}(x_1, \mathrm{J}') = \mathrm{T}\left(: \frac{d\mathscr{L}_\lambda}{dx_1}(x_1) : : \mathscr{L}_\lambda(x_{i_1}) : \ldots : \mathscr{L}_\lambda(x_{i_l}) : \right),$$

$\mathrm{J}' = \{ x_{i_1} \ldots x_{i_l} \}$ and $\bar{\mathrm{T}}(\mathrm{J})$ denotes the antichronologically ordered function

$$\bar{\mathrm{T}}(: \mathscr{L}(x_{j_1}) : \ldots : \mathscr{L}(x_{j_k}) :), \qquad \mathrm{J} = \{ x_{j_1} \ldots x_{j_k} \}.$$

$\mathrm{D}^{(n+1)}$ (by the inductive hypothesis well defined) is the sum of the retarded and advanced $(n + 1)$ point functions, whose support in the distribution sense [4] is contained in the double cone

$$\mathrm{V}_{n+1} = \mathrm{V}_{n+1}^{+} \cup \mathrm{V}_{n+1}^{-}$$

where

$$\mathrm{V}_{n+1}^{+} = (-1)\mathrm{V}_{n+1}^{-}$$

and

$$\mathrm{V}_{n+1}^{-} = \{ x = (x_1, \ldots, x_{n+1}) \in \mathrm{R}^{4(n+1)} ; \; (x_i - x_1)^2 > 0, \quad x_i^0 - x_1^0 \geqslant 0, \quad 1 < i \leqslant n + 1 \} \; [5]$$

One constructs the retarded $(n + 1)$ point function $\mathrm{R}^{(n+1)}$, cutting à la Epstein-Glaser the distribution $\mathrm{D}^{(n+1)}$ at the tip of the double cone V_{n+1}. $\mathrm{T}^{(n+1)}$ can then be expressed as the sum of $\mathrm{R}^{(n+1)}$ plus a bunch of terms well

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(4) In spite of the fact that we interpret operator products $\mathrm{T}^{(n+1)}$ or $\mathrm{D}^{(n+1)}$ as sets of graphs, we might, without problem, refer to distribution properties with respect to the vertices as arguments.

(5) We use the notations $x_i = (x_i^0, x_i^1, \ldots, x_i^3)$, $x_i^2 = (x_i^0)^2 - \Sigma_{j=1}^3 (x_i^j)^2$.

defined through the preceding inductive construction up to the n^{th} order

$$\mathrm{T}^{(n+1)} = \mathrm{R}^{(n+1)} + \sum_{\substack{\mathrm{J}\cup\mathrm{J}'=\{x_2\dots x_{n+1}\}\\ \mathrm{J}\cap\mathrm{J}'=\emptyset}} \bar{\mathrm{T}}(\mathrm{J})\mathrm{T}(x_1, \mathrm{J}')(-1)^{|\mathrm{J}|+1} \equiv \mathrm{R}^{(n+1)} - \mathrm{R}^{(n+1)'} \quad (17)$$

It was shown in Ref. [*6*] that the functions $\mathrm{D}^{(n+1)}$ and $\mathrm{R}^{(n+1)'}$ satisfy Eqs. (14), (15), etc., the time-ordering symbol T being replaced everywhere by D resp. R′. The proof of this fact was carried out in Ref. [*6*] by application of the induction hypothesis to factors $\mathrm{T}^{(i)}$ and $\bar{\mathrm{T}}^{(j)}$ $(i, j < n + 1)$ of the various terms contributing to $\mathrm{D}^{(n+1)}[\mathrm{R}^{(n+1)'}]$. Recollecting terms—taking into account that expressions with an operator $(\square_x + m_i^2)$ acting on a function $\Delta_\pm(x, m_i)$ drop out—one arrives (cf. [*6*]) at relations analogous to (14), (15), and their iterated daughters, from which we quote only the first identity for $\mathrm{D}^{(n+1)}$.

$$\begin{aligned}\mathrm{D}^{(n+1)}\left(:\frac{d\bar{\mathscr{L}}_\lambda}{d\lambda}(x_1)::\bar{\mathscr{L}}_\lambda(x_2):\dots:\bar{\mathscr{L}}_\lambda(x_{n+1}):\right)\\ &= \partial_\mu \mathrm{D}^{(n+1)}\left(:\frac{\delta\bar{\mathscr{L}}_\lambda}{\delta(\partial_\mu(\phi_i + \lambda h_i))}h_i(x_1):\dots\right)\\ &+ \mathrm{D}^{(n+1)}(:(\mathrm{R}_{\lambda;i}\mathrm{A}_{ik}h_k - \underline{h_i(\square + m_i^2)\phi_i})(x_1):\dots)\\ &+ i\sum_{j=2}^{n+1}\delta(x_1 - x_j)\mathrm{D}^{(n)}(:\bar{\mathscr{L}}_{\lambda;\phi_i}h_i(x_j)::\bar{\mathscr{L}}_\lambda(x_2):\\ &\dots(\hat{x}_j)\dots:\bar{\mathscr{L}}_\lambda(x_n):).\end{aligned} \quad (^6)\ (18)$$

The induction hypothesis is recovered for $\mathrm{T}^{(n+1)}$ if one performs the cutting on $\mathrm{D}^{(n+1)}$ in such a way that the chain of relations starting with Eq. (18) can be translated into analogous equalities among R-functions. It is easy to see that this can be achieved by sticking to the recipes already applied in the construction of the two-point function:

α) The cutting has to be carried out separately for the individual terms

$$\mathrm{D}^{(n+1}, \delta^{(4)}(x_1 - x_j)\mathrm{D}^{(n)}, \dots \prod_{\substack{\{i_k\}\\ |\{i_k\}|<n}} \delta^{(4)}(x_1 - x_{i_k})\mathrm{D}^{(2)}$$

appearing in the last link of the chain of equalities starting with Eq. (18). Terms which appeared already at earlier stages of the inductive construction (up to δ-functions) are to be handled as before.

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(6) The hat over x_j shall indicate that this argument is left out.

β) Take care that

$$\mathrm{R}^{(n+1)}\left(: \partial_\mu\left(\frac{\delta \mathscr{L}_\lambda}{\delta(\partial_\mu(\phi_i + \lambda h_i))} h_i\right)(x_1) : \ldots \right) = \partial_\mu \mathrm{R}^{(n+1)}\left(: \frac{\delta \mathscr{L}_\lambda}{\delta(\partial_\mu(\phi_i + \lambda h_i))} h_i(x_1) : \ldots \right)$$

holds.

γ) Add a contact term (arising from a tree graph contraction)

$$\prod_{i=2}^{n+1} \delta^{(4)}(x_1 - x_j) : h_i(\mathrm{B}^n)_{ik} \mathscr{L}_{\lambda;\phi_k}(x_1) :$$

where

$$(\mathrm{B})_{ik} = -\lambda \frac{\partial h_k}{\partial \phi_i}.$$

Remark 1. — It was of crucial importance for our procedure that terms with total derivatives in front vanish in the adiabatic limit and that graphs with an operator $(\square + m^2)$ acting on an external propagator do not contribute on the mass shell. The adiabatic limit, and especially the restriction to the mass shell, require a subtle discussion. We have to ensure that these subtleties do not spoil our deductions.

Epstein and Glaser pointed out that the adiabatic limit and the restriction to the mass shell make sense only after proper mass and wave function renormalization at each stage of their construction. It is easy to see that these renormalization conditions can be fulfilled without destroying our conclusions as they may be imposed first on functions

$$\mathrm{T}\left(: \frac{\delta \mathscr{L}}{\delta(\partial_\mu(\phi_i + \lambda h_i))} \lambda h_i : \ldots \right)$$

and

$$\mathrm{T}(\ldots : \ldots \underline{\varphi} : \ldots)$$

complemented afterwards by a total derivative in front of the first term, and a free field nul operator $(\square + m^2)$ acting on $\underline{\phi}$ in the second term.

Remark 2. — As we require only S-matrix invariance for the point transformation, we have still a great amount of freedom in specifying the off-mass shell behaviour of the $\mathscr{L}_\lambda$ Green's functions. Roughly speaking, we had only to insist that graphs of the same structure either emerging through a series of contractions from another graph or taken as it stands are handled on the same footing. This off-shell indeterminateness in the Green's functions of a theory appearing formally as non-renormalizable has already been observed by Steinman [9] in the context of the renormalization of massive electrodynamics in the Proca gauge.

3. THE ABELIAN HIGGS MODEL

3.1. Preliminaries

The Lagrangian of the abelian Higgs model in t'Hooft's R-gauge (after translation of the physical scalar field) is

$$\begin{aligned}\mathscr{L}_R = & -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}|(\partial_\mu - ieA_\mu)(z + v + i\xi)|^2 - \frac{m^2}{2}z^2 \\ & - \frac{m^2}{2v}z(\xi^2 + z^2) - \frac{m^2}{8v^2}(\xi^2 + z^2)^2 - \frac{1}{2}(\partial_\mu A^\mu + M\xi)^2 + \partial_\mu\phi\partial^\mu\phi^* \\ & - M^2|\phi|^2 - Me\phi^* z\phi, \end{aligned} \tag{19}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$M = ev$, A_μ denotes a vector field with mass M, z stands for a physical scalar particle with mass m, v is the translation parameter, and ϕ and ξ denote the Faddeev-Popov (FP) and the longitudinal ghost, respectively.

To pass over from (19) to the U-gauge formulation of the model, one has to apply to the scalar particle Z and the scalar longitudinal ghost ξ a point transformation

$$(i\xi + z + v) = e^{\frac{i\xi'}{v}}(z' + v) \tag{20}$$

which might be linearly interpolated by the one-parameter family of transformations:

$$\begin{gathered}\xi = \xi_\lambda + \lambda\left(\sin\frac{\xi_\lambda}{v}(z_\lambda + v) - \xi_\lambda\right), \\ z = z_\lambda + \lambda\left(\cos\frac{\xi_\lambda}{v}(z_\lambda + v) - (z_\lambda + v)\right), \\ \xi_{\lambda=0} = \xi, \qquad z_{\lambda=0} = z, \qquad \xi_{\lambda=1} = \xi', \qquad z_{\lambda=1} = z', \\ 0 \leqslant \lambda \leqslant 1 \end{gathered} \tag{20 a}$$

We set:

$$\begin{gathered}\mathscr{L}_\lambda = \mathscr{L}_R(A_\mu, \xi(\xi_\lambda, z_\lambda), z(\xi_\lambda, z_\lambda)), \\ \mathscr{L}_{\lambda=0} = \mathscr{L}_R, \qquad \mathscr{L}_{\lambda=1} = \mathscr{L}_U + \Delta\mathscr{L}. \end{gathered} \tag{21}$$

$\mathscr{L}_U$ denotes the U-gauge Lagrangian with a Stückelberg split [*10*] (7)

$$\begin{aligned}\mathscr{L}_U = & -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}\left|\partial_\mu z + ie\left(A_\mu - \frac{\partial_\mu\xi}{M}\right)(z + v)\right|^2 - \frac{m^2}{2}z^2 \\ & - \frac{m^2}{2v}z^3 - \frac{m^2}{8v^2}z^4 - \frac{1}{2}(\partial_\mu A^\mu + M\xi)^2. \end{aligned} \tag{21 a}$$

(7) For the U-gauge fields $z_{\lambda=1}$, $\xi_{\lambda=1}$ we omit the index λ.

$\Delta\mathscr{L}$ comes out as the result of the point transformation (20) on the FP ghost part and the gauge fixing term of L_R (the latter without the term quadratic in the fields)

$$\begin{aligned}\Delta\mathscr{L} = & -(\partial_\mu A^\mu + M\xi)M\left(\sin\frac{\xi}{v}(z+v) - \xi\right) \\ & - \frac{M^2}{2}\left(\sin\frac{\xi}{v}(z+v) - \xi\right)^2 + \partial_\mu\phi^*\partial^\mu\phi - M^2\,|\,\phi\,|^2 \\ & - Me\phi^*\left(\cos\frac{\xi}{v}(z+v) - v\right)\phi\,. \qquad (21\,b)\end{aligned}$$

To prepare the ground for the subsequent discussion, we make the following remark. The Lagrangians (19) and (21) are first of all synonymous with the corresponding sets of tree graphs. Using the BPHZ renormalization framework we would have to specify a set of counter terms with coefficients worked out as power series in the Planck constant h. However, as we rely on the Epstein-Glaser construction, we start from the Lagrangians (19) and (21) as they stand. That is, m and M are to be considered as the physical masses of the particles ([8]) and v and e are taken as fixed parameters marking the starting point of the inductive Epstein-Glaser construction. Normalization conditions for (R-gauge) one-particle irreducible vertex functions are taken so that they are fulfilled identically by the elementary tree graph vertices at the respective normalization point.

3.2. Mass shell cancellation of $\Delta\mathscr{L}$ contributions

The point transformation (20) does not lead directly to the desired $\mathscr{L}_U$ Lagrangian because of the disturbing term $\Delta\mathscr{L}$. We want to display in this subsection the mechanism which renders the on-shell cancellation of graphs with vertices taken from $\Delta\mathscr{L}$.

We anticipate here one condition, called Stückelber gauge (Sg) invariance, to be imposed on the U-gauge Green's functions

$$T(\vdots\,\bar{\mathscr{L}}_U(x_1)\,\vdots\,\ldots\,\vdots\,\bar{\mathscr{L}}_U(x_n)\,\vdots\,) \qquad (22)$$

where $\bar{\mathscr{L}}_U$ is the interaction part of $\mathscr{L}_U$ [(21 a)], which will play a role in the subsequent discussion. With Sg invariance we mean simply that the physical vector boson field B_μ decomposed for technical reasons by a Stückelberg split [*10*] into

$$B_\mu = A_\mu - \frac{\partial_\mu\xi}{M}$$

can be recovered after renormalization. In other words, one must be able to recollect terms in the Wick expansion of the time-ordered functions (22)

([8]) In the case of the R-gauge Lagrangian (1), we do not insist on a rigorous particle interpretation.

in such a way that fields A_μ and ξ appear only in the factorized combination $:\left(A_\mu - \frac{\partial_\mu \xi}{M} \ \ldots\right):$.

Assuming that the Sg invariant Green's functions (22) have already been constructed, we go on to inspect Green's functions involving $\Delta\mathscr{L}$ vertices. A certain simplification arises from the observation that the sum of fields $(\partial_\mu A^\mu + M\xi)$ does not couple to Sg invariant $\mathscr{L}_U$ vertices. Noting the free field identity

$$\left\langle T\left\{(\partial_\mu A^\mu + M\xi)(x)\left(A_\lambda - \frac{\partial_\lambda \xi}{M}\right)(y)\right\}\right\rangle^f$$
$$= -\partial^\mu_x g_{\mu\lambda}\Delta_f(x-y, M) + \partial_x \Delta_f(x-y, M) = 0 \quad (^9) \quad (23)$$

Taking this into account, we enumerate various contributions to the graph-wise expansion of the $(\mathscr{L}_U + \Delta\mathscr{L})$ Green's functions with vertices taken from the part

$$(\partial_\mu A^\mu + M\xi)M\left(\sin\frac{\xi}{v}(z+v) - \xi\right) \quad (24)$$

of $\Delta\mathscr{L}$.

1) Graphs in which at least one factor $(\partial_\mu A^\mu + M\xi)$ goes into an external propagator line.

2) Graphs with two vertices of type (24) being directly connected exclusively by one propagator factor

$$\langle T((\partial_\mu A^\mu + M\xi)(x)(\partial_\lambda A^\lambda + M\xi)(y))\rangle^f \quad (25)$$

This kind of connection of two vertices was called in Section 2 a tree-like ($t1$) connection.

3) Graphs with a factor (25) between two vertices of type (24) constituting a non-tl connection.

4) Contributions with « ghost loop-like » (gll) subgraphs. By this we mean that several vertices (24) are combined to build up a loop subgraph which might be formally represented as follows.

$$:\left(\frac{-M^2}{v}\right)\left(\cos\frac{\xi}{v}(z+v) - v\right)(x_1):$$
$$\cdot\Delta_f(x_1 - x_2, M)\cdot:\left(-\frac{M^2}{v}\right)\left(\cos\frac{\xi}{v}(z+v) - v\right)(x_2):$$
$$\ldots\cdot:\left(-\frac{M^2}{v}\right)\left(\cos\frac{\xi}{v}(z+v) - v\right)(x_n):\Delta_f(x_n - x_1, M) \quad (26)$$

(9) The free field propagators resp. Wightman functions to be used can be read off from the quadratic terms in (19) or (21 a)

$$\langle T(A^\mu(x)A^\nu(y))\rangle^f = -g^{\mu\nu}\Delta_f(x-y, M) \equiv -\frac{ig^{\mu\nu}}{(2\pi)^4}\int d^4p\,\frac{e^{ip(x-y)}}{p^2 + M^2 + i\varepsilon},$$
$$\langle T(\xi(x)\xi(y))\rangle^f = \Delta_f(x-y, M), \quad \text{etc.}$$

The notion of a gll subgraph was invented to indicate that graphs of the same structure (26) are generated by the FP ghost interaction

$$- \vdots\, \mathrm{M}e\phi^* [\cos \xi/v(z + v) - v]\phi \,\vdots,$$

the connecting Δ_f functions now being considered as FP ghost propagators. It is essential that we work in a gauge where FP ghost are degenerate in mass with the vector boson resp. the longitudinal ghost ξ.

5) The possibilities not yet covered by (1) to (4), namely that factors $(\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)$ are combined either with fields of the mass term

$$- \mathrm{M}^2/2 [\sin \xi/v(z + v) - \xi]^2$$

or with FP ghost loop resp. gll vertices.

One might easily convince oneself that all graphs with vertices from $- (\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi) [\sin \xi/v(z + v) - \xi]$ fall at least in one of the categories (1)-(5). By means of this differentiation of cases, we are now in a position to discuss straightforwardly the cancellation mechanism looked for: contributions of type (1) drop out *per se* on the mass shell because of transversality of the physical vector particles. By contractions of factors

$$\langle \mathrm{T}(\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)(x)(\partial_\lambda \mathrm{A}^\lambda + \mathrm{M}\xi)(y) \rangle^f = (\square + \mathrm{M}^2)\Delta_f(x - y, \mathrm{M}) \sim \delta^4(x - y)$$

to δ-functions in contributions of type (2), a new effective local interaction $\vdots\, \mathrm{M}^2/2 [\sin \xi/v(z + v) - \xi]^2 \,\vdots$ is created, cancelling together with the original mass term $- \mathrm{M}^2/2 [\sin \xi/v(z + v) - \xi]^2$. Terms corresponding to (3) are entirely suppressed (for a justification, see below). Finally, graphs with gll structure (cf. 3.2) vanish together with original ghost loop graphs, remembering that because of the Fermi quantization prescription for FP ghosts each ghost loop carries an extra factor (-1). The list of cancellations is therewith complete. [Contributions of type (5) are subsumed in the discussion of gl and gll graphs on one side, and the original mass term and the one generated by contraction on the other side.] For later reference we remark that the root for the cancellation of gl and gll graphs lies in the simple relation between the part $(\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)(-\mathrm{M}) [\sin \xi/v(z + v) - \xi]$ of the point transformed gauge fixing term and the FP interaction term, as one may write for the latter.

$$- \mathrm{M}e\phi^*\left(\cos \frac{\xi}{v}(z + v) - v \right)\phi = \phi^*\mathrm{M}\frac{\partial}{\partial\xi}\left((-\mathrm{M})\left(\sin \frac{\xi}{v}(z + v) - \xi \right)\right)\phi . \quad (27)$$

We will not give a full proof ([10]) that the preceding formal demonstration

([10]) Which would actually be to a large extent parallel to the argument for the S-matrix invariance of point transformations in Section 2.

of the cancellation of various graphs survives the renormalization procedure, but only sketch the basic ingredients of the argument.

A major part of justification is contained in the trivial remark that graphs of the same structure emerging from different combinations of time-ordered products can (and should) be handled on the same footing. This triviality applies to gl and gll graphs as well as to graphs differing from each other only in as much that one vertex originating from the mass term $-\mathrm{M}^2/2\,[\sin \xi/v(z+v)-\xi]^2$ is replaced by a vertex of the same structure (with an extra minus sign), generated through a contraction of a Feynman propagator to a δ-function.

To justify our treatment of terms with formal factors

$$\langle \mathrm{T}(\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)(x)(\partial_\lambda \mathrm{A}^\lambda + \mathrm{M}\xi)(y) \rangle \tag{28}$$

we note that these terms appear along the way of the inductive renormalization, first at places where they show up as contact terms (i. e. the corresponding time-ordered respectively retarded distribution is concentrated at the point of coincidence of all arguments). Invoking the permissible ambiguities of renormalization, we have the option to keep the terms where a factor (28) constitutes a tl connection and to suppress terms with a non-tl factor (28).

3.3. **Construction of U-gauge green's functions**

The requirements to be fulfilled by the U-gauge Green's functions are the followings:

1) Standard axiomatic principles, that is causality and cutting rules.
2) Proper mass and wave function renormalizations.
3) Sg invariance.
4) A « smooth » high-energy mass shell behaviour, that is high-energy bounds for all scattering amplitudes as good as they can be obtained for any manifestly renormalizable theory.

The construction of Green's functions fulfilling all these points proceeds in several steps. Assume that R-gauge Green's functions saturating requirements (1), (2) and (4) ([11]) above have already been defined. By application of the recipes discussed in Section 2 for the point transformation (20), we arrive at Green's functions of the Lagrangian $\hat{\mathscr{L}} = \mathscr{L}_{\mathrm{U}} + \Delta\mathscr{L}$. In view of the S-matrix invariance of the point transformation, the $\hat{\mathscr{L}}$ Green's functions have a smooth mass shell behaviour.

The necessary mass and wave function renormalizations can be performed without problem (cf. Section 2). Causality constraints and formal cutting

([11]) Condition (4) means in this context that a minimal subtraction scheme has been applied for the renormalization of the R-gauge Green's functions.

rules for the $\hat{\mathscr{L}}$ theory are satisfied automatically, as we implement the point transformation by the Epstein-Glaser procedure, which fulfills these requirements by construction. It remains to be verified that Green's functions involving $\Delta\mathscr{L}$ vertices can be constructed—compatible with the recipes for the point transformation—such that the cancellation mechanism displayed in Section 3.2 applies ([12]) and that the $\bar{\mathscr{L}}_{\mathrm{U}}$ Green's functions obey Sg invariance. The latter condition, together with cutting rules and proper mass and wave function renormalization, render unitarity of the theory.

It is important for the sebsequent discussion to distinguish in $\mathscr{L}_{\mathrm{U}}$ and $\Delta\mathscr{L}$ the parts given by the R-gauge interaction from those emerging through the point transformation, that is, in terms of the interpolating point transformation (20 *a*) the λ independent resp. λ dependent parts.

The point transformation of the kinetic energy terms yields:

$$\begin{aligned}
&\lambda\left\{ \partial_\mu\xi\partial^\mu\left(\sin\frac{\xi}{v}(z+v) - \xi \right) + \partial_\mu z\partial^\mu\left(\cos\frac{\xi}{v}(z+v) - (z+v)\right)\right\} \\
&\quad + \frac{\lambda^2}{2}\left\{\left\langle \partial_\mu\left(\sin\frac{\xi}{v}(z+v) - \xi\right)\right\rangle^2 + \left\langle \partial_\mu\left(\cos\frac{\xi}{v}(z+v) - (z+v)\right)\right\rangle^2\right\} \\
&\quad = \lambda\left\{ \partial_\mu\xi\partial^\mu\left(\frac{\xi z}{v}\right) + \partial_\mu z\partial^\mu\left(-\frac{\xi^2}{2v}\right)\right\} \\
&\quad + \frac{\lambda^2}{2}\frac{(\partial_\mu\xi)^2}{v^2}z^2 + \lambda(1-\lambda)\left(\partial_\mu\xi\partial^\mu\left\langle \sin\frac{\xi}{v}(z+v) - \xi - \frac{\xi z}{v}\right\rangle\right. \\
&\quad \left. + \partial_\mu z\partial^\mu\left\langle \cos\frac{\xi}{v}(z+v) - (z+v) + \frac{\xi}{2v}\right\rangle\right) \qquad (29\,a) \\
&\quad = \frac{(\partial_\mu\xi)^2}{2v^2}(z^2\lambda^2 + 2zv\lambda) + \lambda(1-\lambda)(\ldots) \qquad (29\,b)
\end{aligned}$$

The gauge fixing term $-1/2(\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)^2$ and the additional bilinear part $-ev\mathrm{A}_\mu\partial^\mu\xi$ of (21 *b*) go over into

$$\begin{aligned}
&\lambda\left(\mathrm{A}_\mu\partial^\mu\mathrm{M}\left(\sin\frac{\xi}{v}(z+v) - \xi\right) - (\partial_\mu\mathrm{A}^\mu + \mathrm{M}\xi)\mathrm{M}\left(\sin\frac{\xi}{v}(z+v) - \xi\right)\right) \\
&\qquad\qquad + \frac{\lambda^2}{2}(-\mathrm{M}^2)\left(\sin\frac{\xi}{v}(z+v) - \xi\right)^2, \quad ([13]) \qquad (30)
\end{aligned}$$

and the FP interaction transforms into

$$-\mathrm{M}e\phi*\left(z + \lambda\left(\cos\frac{\xi}{v}(z+v) - (z+v)\right)\right)\phi\,. \qquad (31)$$

([12]) The interplay of the cancellation mechanism due to the point transformation with that of the non-local gauge changing transformation of Section 3.2—both based on contractions of tl connections—might be called « generalized tree graph unitarity ».

([13]) We leave out the bilinear λ independent term.

The remaining terms of $\mathscr{L}_{\mathrm{U}}$ are transformed as follows:

$$\mathrm{V}(\xi, z) \equiv -\frac{m^2}{2}z^2 - \frac{m^2}{2v}z(\xi^2 + z^2) - \frac{m^2}{8v^2}(\xi^2 + z^2)^2 \rightarrow \mathrm{V}(\xi, z) + \lambda\left(\frac{m^2}{2v}z\xi^2 + \frac{m^2}{4v^2}\xi^2 z^2 - \frac{m^2}{8v^2}\xi^4\right) + \lambda^2\frac{m^2}{4v^2}\xi + 0(\lambda(1-\lambda)), \quad (32)$$

$$\mathscr{L}_1(\mathrm{A}_\mu, \xi, z) \equiv e\mathrm{A}_\mu(\partial^\mu z\xi - z\partial^\mu\xi) \rightarrow \mathscr{L}_1 + \lambda\left(e\mathrm{A}_\mu\frac{\partial^\mu\xi}{v}z^2 + \mathrm{MA}_\mu\partial^\mu\left(\sin\frac{\xi}{v}(z+v) - \xi - \frac{\xi z}{v}\right)\right) + 0(\lambda(1-\lambda)), \quad (33)$$

$$\mathscr{L}_2(\mathrm{A}_\mu, \xi, z) \equiv \frac{e^2}{2}\mathrm{A}_\mu^2(z^2 + 2zv + \xi^2) \rightarrow \mathscr{L}_2 - \lambda\frac{e^2}{2}\mathrm{A}_\mu^2\xi^2 + 0(\lambda(1-\lambda)). \quad (34)$$

A special role is played by the terms

$$-\lambda\left(\mathrm{A}_\mu\partial^\mu\left(\mathrm{M}\sin\frac{\xi}{v}(z+v) - \xi\right)\right)$$

and

$$-\lambda\left(\partial_\mu\mathrm{A}^\mu\left(\mathrm{M}\sin\frac{\xi}{v}(z+v) - \xi\right)\right)$$

in (30) which have no counterpart in the general deduction of a point transformation given in Section 2. Taking as an extra rule that the sum of both terms

$$-\lambda\partial_\mu\left(\mathrm{A}^\mu\mathrm{M}\left(\sin\frac{\xi}{v}(z+v) - \xi\right)\right)$$

has to be treated such that the derivative can be pulled out of the respective T product, one assures that these terms vanish in the adiabatic limit and *a fortiori* do not endanger S-matrix invariance.

The elaborate enumeration (29) to (34) becomes useful recalling the net result of Section 2. The S-matrix invariance of the point transformation relies entirely on an appropriate treatment of the part of the point transformed kinetic energy with a linear power of the interpolation parameter λ [i. e. the first term in (29 a)] relative to the point transformed mass term, as this term provides the contractions of tl connections. The only additional rule one has to adhere to is that graphs of the same structure derived from distinct sequences of contractions of tl connections are handled on an equal footing. It is the margin left over by these rules which simplifies considerably the task to construct the ($\mathscr{L}_{\mathrm{U}} + \Delta\mathscr{L}$) Green's functions such that the $\Delta\mathscr{L}$ contributions decouple on mass shell and that the remaining $\mathscr{L}_{\mathrm{U}}$ Green's functions satisfy Sg invariance. Namely, without any reference to specific properties of the R-gauge theory, we can stipulate λ-dependent therms of (29)-(34) relative to each other and to the R-gauge interaction part $1/2e^2\mathrm{A}_\mu^2(\xi^2 + z^2)$ in accordance with the point transformation recipes. As a result of these stipulations, which are enumerated in the following, the renormalized interaction parts of $\mathscr{L}_U + \Delta\mathscr{L}$ carrying more than a linear power of the coupling constant ([14]) will have all desired properties.

i) The R-gauge interaction term

$$\frac{e^2}{2}\left(A_\mu^2(\xi^2 + z^2) - \frac{m^2}{4M^2}(\xi^2 + z^2)^2\right)$$

can be complemented by

$$e^2\left(\frac{(\partial_\mu\xi)^2}{2M^2}z^2 - \lambda A_\mu \frac{\partial_\mu\xi}{M}z^2 - \frac{\lambda}{2}A_\mu^2\xi^2 + \lambda^2\frac{m^2}{8M^2}\xi^4 + \lambda\frac{m^2}{4M^2}\xi^2 z^2\right)$$

picked up from (29 *b*), (32), (33), and (34) so that for $\lambda = 1$ the Sg invariant interaction

$$\frac{e^2}{2} : \left(A_\mu - \frac{\partial_\mu\xi}{M}\right)^2 z^2 - \frac{m^2}{4M^2}z^4 :$$

emerges. Note that this is possible, irrespective of what renormalization prescription has been applied to

$$\frac{e^2}{2}\left(A_\mu^2(\xi^2 + z^2) - \frac{m^2}{4M^2}(\xi^2 + z^2)^2\right)$$

in the R-gauge.

ii) Following the rule spelled out after relation (34), we require for the sum of terms in (30) corresponding to the divergence

$$M(-\lambda)\partial_\mu\left(A_\mu\left(\sin\left(\frac{\xi}{M}e\right)\cdot\left(z + \frac{M}{e}\right) - \xi - \frac{e\xi z}{M}\right)\right) \quad ([15]) \qquad (35)$$

that the derivative has to stand outside the T-products. Furthermore, we dispose of the term

$$M\lambda A_\mu\partial_\mu\left(\sin\left(\frac{\xi e}{M}\right)\left(z + \frac{M}{e}\right) - \xi - \frac{e\xi z}{M}\right)$$

in (33) to vanish identically together with the corresponding part $-\lambda A_\mu\partial^\mu(\ldots)$ of (35).

iii) We have so far not yet constrained the interaction part

$$(\partial_\mu A^\mu)\cdot M\cdot\left(\sin\frac{\xi e}{M}\left(z + \frac{M}{e}\right) - \xi - \frac{e\xi z}{M}\right)$$

([14]) We replace from now in all formulas a v by M/e, to distinguish more clearly different orders in the coupling constant.

([15]) We leave out for the moment the lowest order term $\partial_\mu(A_\mu e\xi z)$.

but only specified other terms in relation to it. We are doing this now, requiring that graphs with tl connections built up by formal factors

$$\vdots\,(-\lambda)\mathrm{M}\left(\sin\frac{\xi}{\mathrm{M}}e\left(z+\frac{\mathrm{M}}{e}\right)-\xi-\frac{\xi z}{\mathrm{M}}\right)(x)$$
$$\vdots\,\langle\,\mathrm{T}(\partial_\mu \mathrm{A}^\mu+\mathrm{M}\xi)(x)(\partial_\mu \mathrm{A}^\mu+\mathrm{M}\xi)(y)\,\rangle^f\,\vdots\,\sin\left(\frac{\xi}{\mathrm{M}}e\right)(\ldots)(y)\,\vdots$$

give rise to an effective local interaction (by contraction of the propagator in between)

$$\vdots\,\frac{\lambda^2}{2}\mathrm{M}^2\left(\sin\frac{\xi}{v}(z+v)-\xi-\frac{\xi z}{\mathrm{M}}\right)^2\vdots$$

which cancels with the corresponding mass term of (30). Graphs with a formal factor

$$\langle\,\mathrm{T}(\partial_\mu \mathrm{A}^\mu+\mathrm{M}\xi),\,(\partial_\mu \mathrm{A}^\mu+\mathrm{M}\xi)\,\rangle^f$$

constituting a non-tl connection, are made to vanish. Note that these rules mean a partial fixation of the term

$$\lambda(\partial_\mu \mathrm{A}^\mu)(-\mathrm{M})\left(\sin\frac{\xi e}{\mathrm{M}}\left(z+\frac{\mathrm{M}}{e}\right)-\xi-\frac{\xi z e}{\mathrm{M}}\right)$$

relative to

$$-\lambda\mathrm{M}^2\xi\left(\sin\frac{\xi}{\mathrm{M}}e\left(z+\frac{\mathrm{M}}{e}\right)-\xi-\frac{\xi z e}{\mathrm{M}}\right)$$

and

$$-\frac{\lambda^2\mathrm{M}^2}{2}\left(\sin\frac{\xi}{\mathrm{M}}e\left(z+\frac{\mathrm{M}}{e}\right)-\xi-\frac{\xi z e}{\mathrm{M}}\right)^2.$$

iv) Ghost loop graphs with vertices from the λ-dependent FP interaction

$$-\lambda\mathrm{M}e\phi^*\left(\cos\left(\frac{\xi}{\mathrm{M}}e\right)\left(z+\frac{\mathrm{M}}{e}\right)-\left(z+\frac{\mathrm{M}}{e}\right)+\frac{\xi^2}{2\mathrm{M}}e\right)\phi$$

have to be defined so that they cancel together with graphs of gll structure (cf. 3.2) built up by the interaction term

$$(\partial_\mu \mathrm{A}^\mu+\mathrm{M}\xi)\left(\sin\frac{\xi e}{\mathrm{M}}\left(z+\frac{\mathrm{M}}{e}\right)-\xi-\frac{\xi z e}{\mathrm{M}}\right)$$

v) The appearance of terms $0[\lambda(1-\lambda)]$ in (29)-(34) has to be considered as a reminiscence to our redundant description of the cancellation mechanism of interaction terms in the $\mathscr{L}_{\mathrm{U}}$ Lagrangian with canonical dimensions higher than four. We require in fact that these terms vanish for $\lambda = 1$.

The redundancy of our procedure can be traced back to the fact that we constructed first the derivative for the set of all Green's functions with respect to the interpolating parameter λ. The actual (non-redundant) cancellation mechanism shows up after integration over λ, leaving aside all trivially cancelling contributions (i. e. cancellations because of algebraic identities before the performance of any contractions). It seems to us rather unlikely that a compact formulation of the cancellation mechanism in non-redundant form is possible. At least we have not been able to find such a formulation. In other words, we had to appeal to the interpolating point transformation in order to bring the whole problem into the range of our technical abilities instead of working directly and exclusively with $\bar{\mathscr{L}}_{\mathrm{U}}$ Green's functions. The latter direct approach would be of course much more attractive.

One could justify altogether stipulations (*i*) to (*v*) invoking the margin of the point transformation recipes. The following discussion of terms in (29)-(34) with a linear power of the coupling constant requires a more detailed argument, as these terms are on one side determined by the role they are supposed to play for generalized tree graph unitarity, and have to fit on the other side into a renormalization scheme for the R-gauge Green's functions with a minimal number of subtractions.

The part of (29)-(34) so far omitted is

$$e\left\{\mathrm{A}_\mu(\partial_\mu z\xi - z\partial_\mu\xi) + \frac{1}{2}\mathrm{A}_\mu^2 z\mathrm{M} - \frac{m^2}{2\mathrm{M}}z(\xi^2 + z^2) - \mathrm{M}\phi^* z\phi \right.$$
$$\left. + \lambda\left\langle -(\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)\xi z - \mathrm{A}_\mu\partial^\mu(\xi z) + \frac{(\partial_\mu\xi^2)^2}{\mathrm{M}}z + \frac{m^2}{2\mathrm{M}}z\xi^2\right\rangle\right\}. \quad (36)$$

We conclude the programme of generalized tree graph unitarity extending the above rules (*i*)-(*v*) to be renormalization of λ-dependent terms in (36):

a) The sum of $-\lambda e\partial_\mu \mathrm{A}^\mu\xi z$ and $-\lambda e\mathrm{A}_\mu\partial^\mu(\xi z)$ has to be treated as a divergence, that is

$$\mathrm{T}(\ldots \vdots - e\lambda(\partial_\mu \mathrm{A}^\mu \xi z + \mathrm{A}_\mu\partial^\mu(\xi z))\vdots) = \partial_\mu \mathrm{T}(\ldots \vdots (-e\lambda)\mathrm{A}_\mu\xi z \vdots)$$

b) One has to insist on cancellations analogous to (*iii*) and (*iv*), namely between the mass term $-1/2\lambda^2\mathrm{M}^2(\xi z)^2$ of (30) and the effective interaction generated by contraction of tl factors

$$\langle \mathrm{T}((\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi), (\partial_\lambda \mathrm{A}^\lambda + \mathrm{M}\xi))\rangle^f$$

on one side, and of gl graphs with vertices from $-Me\phi^* z\phi$ and gll graphs emerging from appropriate combinations of interactions

$$-\lambda e(\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)\xi z\,|_{\lambda=1}$$

(cf. 3.2) on the other side.

c) Rewriting λ-dependent terms of (36), which are quadratic in ξ,

$$e\lambda\left(\frac{(\partial_\mu\xi)^2}{\mathrm{M}}z - \mathrm{M}\xi^2 z - \frac{m^2}{2\mathrm{M}}\xi^2 z\right)$$
$$= e\lambda\left(\partial_\mu\left(\frac{\partial_\mu\xi z}{\mathrm{M}}\right) - \partial_\mu\left(\frac{\xi^2}{2\mathrm{M}}\partial_\mu z\right) - \frac{\xi z}{\mathrm{M}}(\square + \mathrm{M}^2)\xi + \frac{\xi^2}{2\mathrm{M}}(\square + m^2)z\right) \quad (37)$$

one has to require the following properties for the single terms on the right-hand side

$$\mathrm{T}\left(\vdots\, e\lambda\partial_\mu \frac{\partial^\mu \xi z}{\mathrm{M}} \vdots \dots \right) = \partial_\mu \mathrm{T}\left(\vdots\, e\lambda \frac{\partial^\mu \xi z}{\mathrm{M}} \vdots \dots \right),$$

$$\mathrm{T}\left(\dots \vdots\, \lambda e \partial_\mu \left(- \frac{\xi^2}{2\mathrm{M}} \partial^\mu z \right) \vdots \dots \right) = \partial_\mu \mathrm{T}\left(\dots \vdots - \lambda e \frac{\xi^2}{2\mathrm{M}} \partial^\mu z \vdots \dots \right)$$

The terms with free field nul operators $(\square + m^2)z$ resp. $(\square + \mathrm{M}^2)\xi$ in (37) have to provide the contractions of tl connections, which read in formulas

$$\begin{aligned}
&\mathrm{T}\left(\vdots\, \lambda e \left(- \frac{\xi z}{\mathrm{M}} (\square + \mathrm{M}^2)\xi + \frac{\xi^2}{2\mathrm{M}} (\square + m^2) z \right)(x_1) \vdots\, \vdots\, \mathscr{L}^{(2)}(x_2) \vdots \dots \vdots\, \mathscr{L}^{(n)}(x_n) \vdots \right) \\
&\quad = \mathrm{T}\left(\vdots\, \lambda e \left(- \frac{\xi z}{\mathrm{M}} \underline{(\square + \mathrm{M}^2)\xi} + \frac{\xi^2}{2\mathrm{M}} \underline{(\square + m^2) z} \right)(x_1) \vdots \dots \right) \\
&\quad + i \sum_{j=2}^{n} \delta(x_1 - x_j) \mathrm{T}\left(\vdots\, \mathscr{L}^{(2)}(x_2) \vdots \dots \vdots\, \lambda e \left(- \frac{\xi z}{\mathrm{M}} \frac{\delta \mathscr{L}^j}{\delta \xi} \right.\right. \\
&\quad \left.\left. + \frac{\xi^2}{2\mathrm{M}} \frac{\delta \mathscr{L}^j}{\delta z} \right)(x_j) \vdots \dots \vdots\, \mathscr{L}^{(n)}(x_n) \vdots \right)
\end{aligned} \tag{38}$$

where $\mathscr{L}^{(2)}, \dots, \mathscr{L}^{(n)}$ denote any terms of (29)-(34), λ-dependent or not. The underline has the same meaning as before.

Properties *a*) to *c*) can be implemented into the renormalization construction along the lines of (*i*)-(*v*) above. However, we have now to take care that the λ-independent part of (36) (without the FP term) adds up with the λ-dependent one for $\lambda = 1$ to an Sg invariant renormalized interaction:

$$\vdots\, e \left(\mathrm{A}_\mu - \frac{\partial_\mu \xi}{\mathrm{M}} \right)^2 z \cdot \mathrm{M} - \frac{em^2}{2\mathrm{M}} z^3 \vdots .$$

As we have already disposed of the λ-dependent terms in (36), we are lead to the following condition (S) to be fulfilled by the renormalized R-gauge interaction. (S): The renormalized R-gauge interaction

$$\mathscr{L}_{\mathrm{R}}^{(1)} \equiv \vdots\, e \left\{ \mathrm{A}_\mu(\partial^\mu z \xi - z \partial^\mu \xi) + \mathrm{A}_\mu^2 z \cdot \mathrm{M} - \frac{m^2}{2\mathrm{M}} z(\xi^2 + z^2) \right\}$$

can be complemented by a divergence term $\vdots\, (-e)\partial_\mu(\mathrm{A}^\mu \xi z) \vdots$ obeying $\mathrm{T}(\vdots\, (-e)\partial_\mu(\mathrm{A}^\mu \xi z) \vdots \dots) = \partial_\mu \mathrm{T}(\vdots\, (-e)\mathrm{A}^\mu \xi z \vdots \dots)$ such that

$$\begin{aligned}
\mathscr{L}_{\mathrm{R}}^{(1)} - \vdots\, e\partial_\mu(\mathrm{A}^\mu \xi z) \vdots = \vdots\, e &\left\{ \left(\mathrm{A}_\mu - \frac{\partial_\mu \xi}{\mathrm{M}} \right)^2 z\mathrm{M} - \frac{m^2}{2\mathrm{M}} z^3 \right. \\
&+ \frac{\xi z}{\mathrm{M}} (\square + \mathrm{M}^2)\xi - \frac{\xi^2}{2\mathrm{M}} (\square + m^2) z + \partial_\mu \left(- \partial_\mu \xi \frac{\xi z}{\mathrm{M}} + \frac{\xi^2}{2\mathrm{M}} \partial_\mu z \right) \\
&\left. + (\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)\xi z \right\} \vdots ,
\end{aligned} \tag{39}$$

where the first two terms are Sg invariant, the following three satisfy the conditions quoted under *c*), and the last one fits into the cancellation mechanism of gl and gll graphs described in *b*).

Equation (39) holds as a formal algebraic identity. The problem consists in proving that it is true as operator identity, and especially that the various terms on the right-hand side of (39) have the desired properties. Instead of giving a direct proof for this ([16]) we show that condition (S) is equivalent with the statement of renormalized Slavnov identities proven rigorously by BRS [*3*]. To quote the Slavnov identities we have to introduce some new notations. Let

$$z(\mathrm{J}, \eta) = \mathrm{T} \exp\left(i \int d^4x(\mathscr{L}_{\mathrm{R}} + \mathrm{J}\phi + \phi^*\eta + \phi\eta^*)\right)$$

be the generating functional of the R-gauge Green's functions, where $\mathrm{J}\cdot\phi$ is a shorthand for the source terms $\mathrm{J}_\mu \mathrm{A}^\mu + \mathrm{J}_1 \cdot z + \mathrm{J}_2 \cdot \xi$ and η, η^* are the anticommuting sources of the FP ghosts. With $\mathrm{C}(\mathrm{J}, \eta) = -i \ln z(\mathrm{J}, \eta)$ we denote the generating functional of the connected Green's functions. The Slavnov identities are given in functional form by (cf. [*3*] and [*11*]):

$$\mathrm{T} \int d^4x\eta^*(x)\left(\partial^x_\mu \frac{\partial}{\delta \mathrm{J}^\mu(x)} + \mathrm{M} \frac{\delta}{\delta \mathrm{J}_2(x)}\right)\mathrm{C}(\mathrm{J}, \eta)$$

$$= \mathrm{T} \int d^4x(\mathrm{J}^\mu \partial^x_\mu - e\mathrm{J}_1(\xi + v) + e\mathrm{J}_2 z) \frac{\delta}{\delta\eta(x)} \mathrm{C}(\mathrm{J}, \eta) \quad ({}^{17}) \quad (40)$$

We show first that condition (S) implies Slavnov identities. To start with we demonstrate our pedestrian procedure at the trivial example of tree graphs in lowest order of the coupling constant. That is, we consider the expression:

$$\mathrm{T}^{(1)} = i \int d^4x' \{ \mathrm{T}(\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)(x) : \mathscr{L}^{(1)}_{\mathrm{R}}(x') : \}^t \quad ({}^{18}) \qquad (41)$$

Adding to $:\mathscr{L}^{(1)}_{\mathrm{R}}:$ the divergence $-e\partial_\mu(\mathrm{A}^\mu \xi z)$ — by integration over x' it drops out again — and decomposing the sum according to (39), we replace in (41) $:\mathscr{L}^{(1)}_{\mathrm{R}}:$ by

$$e : \left(\mathrm{A}_\mu - \frac{\partial_\mu \xi}{\mathrm{M}} \right)^2 \cdot z \cdot \mathrm{M} - \frac{m^2}{2\mathrm{M}} z^3 + \frac{\xi z}{\mathrm{M}} (\Box + \mathrm{M}^2)\xi - \frac{\xi^2}{2\mathrm{M}} (\Box + m^2) z$$

$$+ \partial_\mu \left(- \partial^\mu \xi \frac{\xi z}{\mathrm{M}} + \frac{\xi^2}{2\mathrm{M}} \partial^\mu z \right) - (\partial_\mu \mathrm{A}^\mu + \mathrm{M}\xi)\xi z : .$$

([16]) We know a direct proof within the Epstein-Glaser renormalization scheme which, we believe, does not lead to any additional insight.

([17]) Equation (40) represents the formal (i. e. unrenormalized) version of the Slavnov identities.

([18]) We use the notations of Section 2.

The first two terms do not contribute to (41), as $(\partial_\mu A^\mu + M\xi)$ « screens » Sg invariant interactions:

$$\left\langle T\left(\partial_\mu A^\mu + M\xi, A_\lambda - \frac{\partial_\lambda \xi}{M}\right)\right\rangle^f \equiv 0 .$$

Omitting the divergence

$$:\partial_\mu\left(-\frac{\xi z}{M}\partial^\mu\xi + \frac{\xi^2}{2M}\partial^\mu z\right):$$

we distinguish graphwise contributions from the remaining terms with factors

$$F_1(x - x') = \langle T(\partial_\mu A^\mu + M\xi)(x), (\partial_\lambda A^\lambda + M\xi)(x') \rangle^f ,$$
$$F_2(x - x') = \langle T(M\xi(x), (\Box + M^2)\xi(x')) \rangle^f$$

and

$$\langle T(M\xi(x), \xi(x') \rangle^f = M\Delta_f(x - x', M)$$

respectively. Contributions with factors F_1 and F_2 cancel out (as $F_1 = F_2$). We get:

$$T^1 = i\int d^4x'\Delta_f(x - x', M)\left\{ Me : \frac{z}{M}(\Box + M^2)\xi - \frac{\xi}{M}(\Box + m^2)z - (\partial_\mu A^\mu + M\xi)z(x') : \right\}^t . \quad (42)$$

One might easily verify by graphwise comparison that Eq. (42) phrases the Slavnov identity (40) to lowest order in the coupling constant. The first two terms on the right-hand side of (42), in which an external z resp. ξ propagator leg is contracted into the vertex ([19]) by virtue of the operator $(\Box + M^2)\xi$ (resp. $(\Box + m^2)z$, represent the non-linear part

$$T\int d^4x(-eJ_1\xi + eJ_2 z)\frac{\delta}{\delta\eta(x)}C(J, \eta)$$

of (40), whereas the linear part

$$T\int d^4x(J^\mu\partial_\mu^x - eJ_1 v)\frac{\delta}{\delta\eta(x)}C(J, \eta)$$

arises from the last term

$$i\int d^4x'\Delta_f(x - x', M)\{(-Me) : (\partial_\mu A^\mu + M\xi)z(x') : \}^t .$$

The strategy we follow to derive from (S) Slavnov identities, to all orders of perturbation theory, consists in a successive exploitation of properties

([19]) We did not specify the external fields in Eq. (42). They are subsumed in the notation $\{\ \}^t$ = set of all corresponding tree graphs.

[collected in (S)] of the renormalized interaction term $\mathscr{L}_R^{(1)}$, as it was done just before for the lowest order tree graph Green's functions.

Let $T^{n,C}[(\partial_\mu A^\mu + M\xi)(x), X]$ be the set of all connected graphs contributing in n^{th} order of the coupling constant to the Green's function of the fields $(\partial_\mu A^\mu + M\xi)$ and X, where X denotes any collection of fields A_μ, ξ, z. We only discuss the case that no external FP ghost fields are present. The more general case can be handled along the same lines, modulo a tedious but straightforward combinatorical argument, which takes the Fermi statistics of the FP ghosts into account.

We set up an iterative chain of manipulations [justified by hypothesis (S)] recasting therewith successively $T^{n,c}$:

1) We single out from $T^{n,c}$ those graphs in which $(\partial_\mu A^\mu + M\xi)(x)$ is connected with an $\mathscr{L}_R^{(1)}$ vertex ([20]) and rearrange them so that the $\mathscr{L}_R^{(1)}$ vertices under consideration appear as being derived from the interaction [equivalent to $\mathscr{L}_R^{(1)}$]

$$:e\left\{\frac{1}{2}\left(A_\mu - \frac{\partial_\mu\xi}{M}\right)^2 z\cdot M - \frac{m^2}{2M}z^3 + \frac{\xi z}{M}(\Box + M^2)\xi \right. \\ \left. - \frac{\xi^2}{2M}(\Box + m^2)z - (\partial_\mu A^\mu + M\xi)\xi z\right\}:. \quad (43)$$

The first two terms in (43) can be ignored because the combination of fields $(\partial_\mu A^\mu + M\xi)$ does not couple to Sg invariant interactions. The third and fourth terms provide the contractions of tl connections.

2) What concerns vertices originating from the last term $-e(\partial_\mu A^\mu + M\xi)\xi z$ we distinguish as to what type of vertex resp. field the factor $(\partial_\mu A^\mu + M\xi)$ is connected:

i) to the external field $(\partial_\mu A^\mu + M\xi)(x)$;
ii) to another external field;
iii) to an $\mathscr{L}_R^{(1)}$ vertex;
iv) to an $\mathscr{L}_R^{(2)}$ vertex where

$$\mathscr{L}_R^{(2)} \equiv :\frac{e^2}{2}A_\mu^2(\xi^2 + z^2) - \frac{m^2}{8M^2}(\xi^2 + z^2)^2:$$

Contributions of type *i*) drop out together with graphs in which the factor $(\Box + M^2)\xi$ of the third term in (43) is connected with $(\partial_\mu A^\mu + M\xi)(x)$. In both cases the propagator leg starting at x is contracted into vertices of graphs with identical structure and a factor (-1) relative to each other.

3) Contributions of type *iii*) are once more rearranged in such way that the second $\mathscr{L}_R^{(1)}$ vertex appears to be derived from the $\mathscr{L}_R^{(1)}$ equivalent

([20]) This means, more properly stated, that we single out those terms in the Wick expansion of the Green's function, where $(\partial_\mu A^\mu + M\xi)$ is contracted into a propagator together with a field of an interaction term $\mathscr{L}_R^{(1)}$.

interaction (43). Repeating the argument given in (2), we may state that graphs, in which the second $\mathscr{L}_R^{(1)}$ vertex originates from the Sg invariant part of (43) vanish. In graphs with vertices from the third and fourth term in (43) we perform all possible contractions of tl connections, starting at the second generation of $\mathscr{L}_R^{(1)}$ vertices. We close finally the first circle of the iterative procedure by distinguishing for vertices originating from the last term in (43) the possibilities as to what connection the field $(\partial_\mu A^\mu + M\xi)$ enters into.

The effect of these manipulations is that, starting at the point x of the field $(\partial_\mu A^\mu + M\xi)$ successively all possible ghost-like propagator paths are built up, which means that step by step the contributions to the right-hand side of the Slavnov identities are generated. Contractions of external lines by operators $(\Box + m^2)$ and $(\Box + M^2)$ appearing in the $\mathscr{L}_R^{(1)}$ equivalent interaction terms

$$e\frac{\xi z}{M}(\Box + M^2)\xi$$

and

$$-\frac{\xi^2}{2M}(\Box + m^2)z$$

respectively produce the non-linear part

$$T\int d^4x(-eJ_1\xi + eJ_2z)\frac{\delta}{\delta\eta(x)}C(J, \eta)$$

of the right-hand side of the Slavnov identities, whereas the linear part

$$T\int d^4x(J^\mu\partial_\mu^x - eJ_1v)\frac{\delta}{\delta\eta(x)}C(J, \eta)$$

arises from connections of external fields with factors $(\partial_\mu A^\mu)$ resp. $M\xi$ of the interaction parts $e(\partial_\mu A^\mu + M\xi)\xi z$. Contributions from contractions of internal tl connections to be performed at the intermediate steps of the iterative procedure cancel out by an interplay of $\mathscr{L}_R^{(1)}$ and $\mathscr{L}_R^{(2)}$ vertices according to the following scheme: Graphs with an $\mathscr{L}_R^{(1)}$ vertex, connected on one side with a factor $(\partial_\mu A^\mu + M\xi)$, and on the other side contracted into a second $\mathscr{L}_R^{(1)}$) vertex—by either an operator $(\Box + m^2)$ resp. $(\Box + M^2)$ acting on the tl connecting line in between or a tl propagator factor

$$\langle T(\partial_\mu A^\mu + M\xi), (\partial_\lambda A^\lambda + M\xi) \rangle^f$$

vanish together with graphs, in which an $\mathscr{L}_R^{(2)}$ vertex is connected with a factor $(\partial_\mu A^\mu + M\xi)$. Graphs with vertices from the $\mathscr{L}_R^{(1)}$ term

$$:\frac{e\xi z}{M}(\Box + M^2)\xi - \frac{e\xi^2}{2M}(\Box + m^2)z:$$

where the factors $(\Box + m^2)\xi$ and $(\Box + m^2)z$ yield a contraction into an

$\mathcal{L}_{R}^{(2)}$) vertex and one field ξ in front is connected with a factor $(\partial_\mu A^\mu + M\xi)$ —that is, effectively $M\xi$— drop out identically.

Remark. — The non-appearance of ghost loop graphs in our deductions is justified by the hypothesis we started from, namely that gl and gll graphs cancel out.

We recall that the preceding graphological discussion has its solid basis in the framework of the Epstein-Glaser renormalization construction. To contract tl connections to δ-functions and to suppress all terms, in which formally appear contractions of non-tl connections, is compatible with causality and unitarity (cf. Sections 2 and 3.2). The only additional element of our argumentation was the nearly tautological principle: formally equal graphs are equal.

To see that the content of the Slavnov identities is in fact identical with (S), we appeal once more to the inductive character of the Epstein-Glaser construction. The statement is evidently true in tree graph approximation. The same holds for the absorptive part of one-particle irreducible one-loop graphs as they are built up by two tree graph chains glued together through free field Wightman functions. That is, the Slavnov identity [as well as (S)] is recovered for the absorptive one-loop contributions for the unique reason that one can manipulate the vertices of the tree graph chains in the way described in (S).

The full time-ordered amplitude is obtained from the absorptive part via a generalized dispersion relation ([21]). The latter operation is compatible with Slavnov identities only if the possibility to rearrange terms according to (S) is left untouched. In fact, considering the time-ordered amplitudes as distributions whose arguments are the interaction vertices, one might formulate condition (S) as well as the Slavnov identities as equations between distributions. We know already that away from the point where all arguments coincide (the « coincidence point »), the equalities expressing (S) and Slavnov identities are equivalent and satisfied because the statement holds for the absorptive part. It follows that the dispersion relation rendering the extension (and thereby the proper definition) of the time-ordered distributions to the coincidence point ([22]) preserves the set of equalities expressing Slavnov identities if it preserves the equivalent identities standing for (S).

This concludes the proof for the one-loop order.

The general inductive step to pass over from n to $(n + 1)$ loop order can be done using analogous arguments. One first verifies that the assertion

([21]) Epstein and Glaser use actually not the full absorptive part, but D = R + A (cf. Section 2). Applying a dispersion relation means in their language the « cutting » of distribution D. We commit a slight abuse of language using the word « absorptive » part synonymously with D.

([22]) That is, the time-ordered distribution is elsewhere uniquely determined through the absorptive part.

is true as identity between distributions away from the coincidence point by applying the induction hypothesis to lower order Green's functions [i. e. lower than $(n+1)^{\text{th}}$ loop order] glued together through free field Wightman functions. One concludes then that the extension to the coincidence point with help of a generalized dispersion relation must necessarily be compatible with condition (S) in order that the Slavnov identities are satisfied.

4. THE NON-ABELIAN HIGGS-KIBBLE MODEL

In t'Hooft's R-gauge the model under consideration is given by [12]

$$\begin{aligned}
\mathscr{L}_{\mathrm{R}} = & -\frac{1}{4}(\vec{\mathrm{B}}_{\mu\nu})^2 + \frac{1}{4}\,\mathrm{tr}\,(\mathrm{D}_\mu \mathrm{C}(\mathrm{D}^\mu \mathrm{C})^*) - \frac{m^2}{2}z^2 - \frac{m^2}{2\sqrt{2}\mathrm{F}}z(z^2+\vec{\mathrm{K}}^2) \\
& - \frac{m^2}{16\mathrm{F}^2}(z^2+\vec{\mathrm{K}}^2)^2 - \frac{1}{2}(\partial_\mu \vec{g}^\mu + \mathrm{M}\vec{\mathrm{K}})^2 + \partial_\mu \vec{\phi}^* \partial^\mu \vec{\phi} - \mathrm{M}^2 \vec{\phi}^* \vec{\phi} \\
& + g\partial_\mu \vec{\phi}^*(g^\mu \otimes \vec{\phi}) - \frac{\mathrm{M}g}{2}\vec{\phi}^*(z+\vec{\mathrm{K}}\otimes)\vec{\phi}, \\
& \vec{\mathrm{B}}_{\mu\nu} = \partial_\mu \vec{g}_\nu - \partial_\nu \vec{g}_\mu + g\,\vec{g}_\mu \otimes \vec{g}_\nu, \qquad (44) \\
& \mathrm{M} = g\cdot\frac{\mathrm{F}}{\sqrt{2}}, \\
& \mathrm{D}_\mu \mathrm{C} = \left(\partial_\mu - i\frac{g}{2}\vec{g}_\mu\cdot\underline{\tau}\right)((z+\sqrt{2}\mathrm{F})\mathbb{1} + i\vec{\mathrm{K}}\cdot\tau), \\
& \vec{\mathrm{K}}\cdot\underline{\tau} = i\Sigma \mathrm{K}_i\cdot\tau_i .
\end{aligned}$$

We use here the following notations: τ = Pauli matrices, $\mathbb{1}$ = unit matrix, $(\vec{x}\otimes\vec{y})^i = \varepsilon^{ijk}x_j y_k$, F is the translation parameter of the spontaneous symmetry breaking, z denotes a physical scalar particle, $\vec{\mathrm{K}}$ and $\vec{\phi}$ stand for the longitudinal and FP ghosts, respectively, and $\vec{g}_\mu$ is an SU(2) triplet of gauge fields.

To pass over to the U-gauge we perform the following substitutions in (44)

$$((z+\sqrt{2}\mathrm{F})\mathbb{1} + i\vec{\mathrm{K}}\cdot\underline{\tau}) = e^{\frac{i\vec{\mathrm{K}}'\cdot\underline{\tau}}{\sqrt{2}\mathrm{F}}}(z'+\sqrt{2}\mathrm{F})\mathbb{1}\,, \tag{45}$$

$$\frac{i\vec{g}_\mu\cdot\underline{\tau}}{2} = e^{\frac{i\vec{\mathrm{K}}'\cdot\underline{\tau}}{\sqrt{2}\mathrm{F}}}\left(\frac{i}{2}\right)\underline{\tau}\left(\vec{g}'_\mu - \frac{\partial_\mu \vec{\mathrm{K}}'}{\mathrm{M}}\right)e^{-i\frac{\vec{\mathrm{K}}'\cdot\underline{\tau}}{\sqrt{2}\mathrm{F}}} + \frac{1}{g}\left(\partial_\mu e^{\frac{i\vec{\mathrm{K}}'\cdot\underline{\tau}}{\sqrt{2}\mathrm{F}}}\right)e^{-i\frac{\vec{\mathrm{K}}'\cdot\underline{\tau}}{\sqrt{2}\mathrm{F}}}, \tag{46}$$

$$\begin{aligned}
& \mathscr{L}_{\mathrm{R}} = \mathscr{L}_{\mathrm{R}}(\vec{g}_\mu, \vec{\mathrm{K}}, z), \\
& \mathscr{L}_{\mathrm{R}}(\vec{g}_\mu(\vec{g}'_\mu, \vec{\mathrm{K}}', z'), \vec{\mathrm{K}}(\vec{\mathrm{K}}', z'), z(\vec{\mathrm{K}}', z') \equiv \mathscr{L}_{\mathrm{U}}(\vec{g}'_\mu, \vec{\mathrm{K}}', z') + \Delta\mathscr{L} \qquad (47)
\end{aligned}$$

where $\mathscr{L}_U$ denotes the U-gauge Lagrangian with a Stückelberg split.

$$\mathscr{L}_U = -\frac{1}{4}(\vec{B}'_{\mu\nu})^2 + \frac{1}{2}\left|\partial_\mu z' + \frac{ig}{2}\left(\vec{g}'_\mu - \frac{\partial_\mu \vec{K}'}{M}\right)(z' + \sqrt{2}F)\right|^2$$
$$- \frac{m^2}{2}z^2 - \frac{m^2}{2\sqrt{2}F}z^3 - \frac{m^2}{16F^2}z^4 - \frac{1}{2}(\partial_\mu \vec{g}'^\mu + M\vec{K}')^2 \quad (48)$$

$\Delta\mathscr{L}$ stands for the point-transformed FP and gauge fixing terms of (44)

$$\Delta\mathscr{L} = [FP] - \frac{1}{2}(\partial_\mu \vec{g}^\mu(g'_\mu, \vec{K}') + M\vec{K}(\vec{K}', z))^2 + \frac{1}{2}(\partial_\mu g^\mu + M\vec{K}')^2, \quad (49)$$

where

$$[FP] = \partial_\mu \vec{\phi}^* \partial^\mu \vec{\phi} - M^2 |\vec{\phi}|^2 + g\partial_\mu \vec{\phi}^*(\vec{g}^\mu(\vec{g}'_\mu, \vec{K}') \otimes \vec{\phi})$$
$$- \frac{Mg}{2}\vec{\phi}^*((z(\vec{K}', z') + \vec{K}(\vec{K}', z') \otimes)\vec{\phi}). \quad (50)$$

The arguments for the mass shell equivalence of the R- and U-gauge formulations are to a great part parallel to those used for the abelian model of Section 3. The S-matrix invariant point transformation (45), (46) applied to a minimally subtracted R-gauge theory renders a smooth mass shell behaviour of the $(\mathscr{L}_U + \Delta\mathscr{L})$ Green's functions. To display (formally) the on-shell decoupling of $\Delta\mathscr{L}$ contributions, one might exploit as before (Section 3.2) the Sg invariance of the $\mathscr{L}_U$ Green's functions and follow up the cancellations first among vertices originating from the part $- M^2/2\vec{K}^2(\vec{K}', z')$ of (49) and vertices generated by contractions

$$\langle T(\partial_\mu g'^{\mu,i} + MK'^i)(x), (\partial_\lambda g^{\lambda,j} + MK'^j)(y) \rangle \sim \delta^{ij}\delta^{(4)}(x - y)$$

and second, among original ghost loop (gl) graphs and graphs with ghost loop-like (gll) structure (cf. Section 3). The latter point requires some further element of argumentation, which has no counterpart in the abelian case.

The problem is that the point-transformed FP interaction (50) is not yet in a shape such that the cancellation of original ghost loop and gll graphs is obvious. In the abelian case the analogous cancellation was derived from the simple relation [Eq. (27)] between the FP interaction and the point-transformed gauge fixing term. To achieve an analogous relation in the present non-abelian model, we introduce an additional point transformation of the FP ghosts.

Let A_{ik} be the matrix of the non-linear chiral transformation

$$e^{i\frac{\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}} \to e^{i\frac{\vec{\mathscr{L}}\cdot\underline{\tau}}{2}} e^{i\frac{\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}} = e^{i\frac{\vec{K}''\cdot\underline{\tau}}{\sqrt{2}F}}$$

$$A_{ik} = \left.\frac{\partial K''_i}{\partial \mathscr{L}_k}\right|_{\mathscr{L}=0}$$

The FP interaction can thereby be rewritten as follows

$$[\mathrm{FP}] = \partial_\mu \phi^{i*}\left(\left(\frac{\delta}{\delta(\partial_\mu \mathrm{K}'_j)} - \frac{\delta}{\delta(g'_{\mu,j})}\right)g^{\mu,i}\right)\mathrm{A}_{jk}\partial_\mu\phi^k + \mathrm{M}\partial_\mu\phi^{i*}\frac{\delta \mathrm{A}^{\mu i}}{\delta \mathrm{K}'_j}\mathrm{A}_{jk}\phi^k - \mathrm{M}^2\phi^{i*}\frac{\delta \mathrm{K}^i}{\delta \mathrm{K}'_j}\mathrm{A}_{jk}\phi^k \quad (51)$$

Equation (50) suggests a point transformation of the FP ghosts:

$$\phi^{i*} = \tilde{\phi}^{i*}, \qquad \tilde{\phi}^i = \mathrm{A}_{ik}\phi_k \quad (52)$$

which translates (50) into

$$[\mathrm{FP}] = \partial_\mu \tilde{\phi}^{i*}\left(\left(\frac{\delta}{\delta(\partial_\mu \mathrm{K}'_j)} - \frac{\delta}{\delta(g'_{\mu,j})}\right)g^{\mu i}\right)\partial_\mu \tilde{\phi}^j - \mathrm{M}^2\tilde{\phi}^{i*}\frac{\partial \mathrm{K}_i}{\delta \mathrm{K}'_j}\tilde{\phi}^j + \mathrm{M}\partial_\mu\tilde{\phi}^{i*}\frac{\delta g^{\mu i}}{\delta \mathrm{K}'_j}\tilde{\phi}_j \quad (53)$$

and $\Delta\mathscr{L}$ into

$$\Delta\mathscr{L} = [\mathrm{FP}](\tilde{\phi}^*, \tilde{\phi}, \mathrm{K}', z') - \frac{1}{2}(\partial_\mu \vec{g}^\mu + \mathrm{M}\vec{\mathrm{K}})^2 + \frac{1}{2}(\partial_\mu \vec{g}'^\mu + \mathrm{M}\vec{\mathrm{K}}')^2. \quad (54)$$

From (53) and (54) one can read off the relation analogous to (27) between the FP interaction and the point-transformed gauge fixing term rendering the structural equality of gl and gll graphs.

The proof for the equivalence of Slavnov identities and the condition to be imposed on the R-gauge Green's functions [the analogon of condition (S) in Section 3.3] in order that the point transformation (45), (46) leads to a respectable U-gauge theory proceeds along the same lines as in the abelian case. The additional point transformation (54) is reflected in the Slavnov identities by the appearance of a term non-linear in the ghost fields having no analogon in the abelian case (cf. [3]):

$$\frac{1}{2}\mathrm{T}\int d^4x(\vec{\phi}^* \otimes \vec{\phi}^*)\vec{\eta}(x)\mathrm{C}(\mathrm{J}, \vec{\eta}),$$

where C denotes as before the generating functional of the connected Green's functions and $\vec{\eta}$ stands for the source function of the FP ghost $\vec{\phi}^*$.

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