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Crossing and experimental ππ S and P waves

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Crossing and experimental $\pi\pi$ S and P waves

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ABSTRACT. — We test the crossing properties of fits to experimental s-and p-wave $\pi\pi$ phases of Baton *et al.* [1], using the equations of Roy [2], obtained from fixed t dispersion relations. A sum rule for the scattering lengths a_0 and a_2 is also used, in which the asymptotic contribution is an important ingredient. Then, our main conclusions are that :

a. the method appears to be very precise and useful;

b. if one believes strictly in the δ_0^2 of Baton the scattering lengths disagree with Weinberg's predictions but a_0 agrees with the recent Ke^4 indications;

c. if δ_0^2 is lower than Baton's values, acceptable agreement with Weinberg's predictions can be achieved.

RÉSUMÉ. — On teste les propriétés de croisement de fits à des déphasages $\pi\pi$ expérimentaux de Baton et coll. [1] pour les ondes s et p, en utilisant les équations de Roy [2] obtenues par des relations de dispersion à t fixé. Une règle de somme pour les longueurs de diffusion a_0 et a_2 est aussi utilisée, dans laquelle la contribution asymptotique est un ingrédient important. Nos conclusions principales sont alors que :

a. la méthode s'avère très précise et utile;

b. si l'on croit strictement au δ_0^2 de Baton les longueurs de diffusion sont en désaccord avec les prédictions de Weinberg, mais a_0 est en accord avec les récentes indications de Ke^4;

c. si δ_0^2 est plus bas que les valeurs de Baton, un accord acceptable avec les prédictions de Weinberg peut être obtenu.

Recently, Roy has written equations for $\pi\pi$ partial wave amplitudes [2] starting from fixed momentum transfer dispersion relations. These equations express the crossing and analyticity properties of the $\pi\pi$ amplitude directly in terms of *physical region* amplitudes, and this fact makes them very interesting for performing low energy $\pi\pi$ phenomenology, as we have argued elsewhere [3].

In this note we report the results of the following investigation. We want to know to what extent the existing data on $\pi\pi$ scattering [1] allow to determine the values of the s-waves scattering lengths, if one uses Roy's equations to perform the analytic continuation down to threshold.

Roy's crossing equations for physical partial wave amplitudes are of the form [3] :

$$(1) \qquad f_l^{\mathrm{I}}(s) = \mathrm{S.\,T.} + \sum_{\mathrm{I}'=0}^{2} \sum_{l'=0}^{\infty} (2\,l'+1) \times \int_4^{\infty} ds'\, \mathrm{G}_{l,\mathrm{I}}^{l',\mathrm{I}'}(s, s')\, \mathcal{I}m\, f_{l'}^{\mathrm{I}'}(s') \qquad (m_\pi = 1).$$

The subtraction terms S. T. are present in s-waves and p-wave, they are linear polynomials in s whose coefficients are determined by the two scattering lengths a_0 and a_2. The explicit form of Equation (1) is given in Reference [3]. As shown in reference [3], it is a good approximation for $s \lesssim m_\rho^2$ to retain only the contributions of s- and p-wave absorptive parts in the right hand side of Equation (1). Also, the kernels G (s, s') are highly convergent in s' and one may restrict the integration range to $s' \lesssim 115$. We concentrate on the s- and p-wave equations ($l = 0, 1$) and require consistency between RHS and LHS of Equation (1) at low energies ($s < m_\rho^2$).

Good experimental $\pi\pi$ s- and p-wave phase shifts are available between 500 and 1000 MeV [1] (with the well-known up-down ambiguity in $\mathrm{I} = 0$). Our aim is to continue the phases down to threshold by some flexible form and to fix the parameters through the consistency relations (1).

Here, the imaginary part of the $\mathrm{I} = 1$ p-wave is parametrized as in Reference [4] (the real part being determined by elastic unitarity). This has the advantage of producing an excellent fit to experiment (with $m_\rho = 767$ MeV and $\Gamma_\rho = 135$ MeV) for a large range of value of the arbitrary parameter a_1 — the p-wave scattering length —.

We choose for the $\mathrm{I} = 0,2$ s-waves the unitary form

$$(2) \qquad f_0^{\mathrm{I}}(s) = \left[\sqrt{\frac{s-4}{s}}\,\mathrm{cotg}\,\delta_0^{\mathrm{I}}(s) - i\sqrt{\frac{s-4}{s}}\right]^{-1}, \qquad \mathrm{I} = 0, 2$$

and the K-matrix is parametrized as follows.

(i) In the experimental range $\bar{s}_I \leqq s \leqq 50$ ($\bar{s}_I \gtrsim 13$) we use the fits to the experimental results of Baton *et al.* [1] (either « up » or « down » solution for I = 0) (see *fig.* 1);

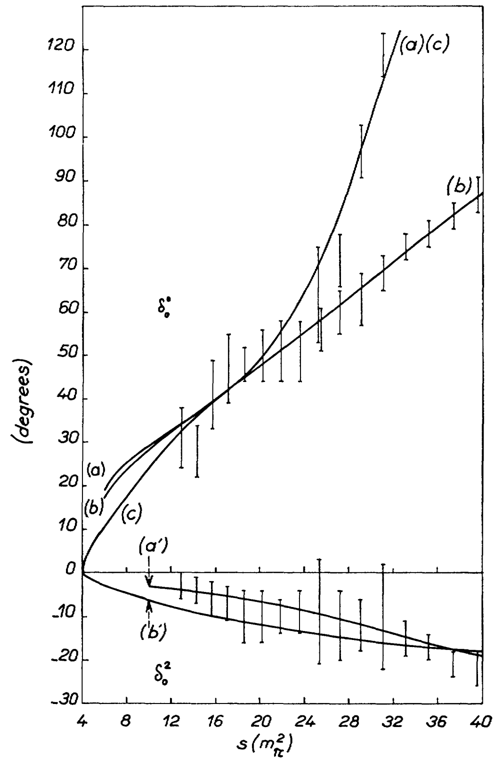

Fig. 1. — S-wave $\pi\pi$ phase-shifts (Experimental data are from [1]) :
I = 0 : Curve (a) [resp. (b)] : fit to experimental data for the « up » [resp. « down »] δ_0^0 phase-shift;
Curve (c) : phase-shift corresponding to the curves of figure 2.
I = 2 : Curve (a') : fit to experimental data;
Curve (b') : phase-shift corresponding to the curves of figure 2.

(ii) For $s > 50$ we use the continuation of the previous expression-this does not play a crucial role, owing to quick convergence of the s' integration;

(iii) In the low energy region $4 \leqq s \leqq \bar{s}_I$ we use the flexible form

$$(3) \quad \sqrt{\frac{s-4}{s}} \operatorname{cotg} \delta_0^I = \frac{A^I + B^I s}{1 + C^I s} + \frac{2}{\pi}\sqrt{\frac{s-4}{s}} \operatorname{Log}\left(\frac{\sqrt{s-4}+\sqrt{s}}{2}\right).$$

The joining of this expression to the fit to experimental data for $s \geq \bar{s}_I$ is realized by requiring continuity of the expression and of its derivative at $s = \bar{s}_I$; this determines two of the three parameters A^I, B^I, C^I in each channel. We have noticed that $\bar{s}_0$, the joining point in $I = 0$, is an important parameter, while the results are rather insensitive to $\bar{s}_2$ which was taken to be $\bar{s}_2 = 16$. In letting $\bar{s}_0$ vary, we allow ourselves to neglect the first few $I = 0$ experimental points which may have small statistical significance [1]. At this stage, we have four parameters : the three s- and p-wave scattering lengths a_0, a_2, a_1, and the position $\bar{s}_0$ of the joining point in $I = 0$.

For each set $(a_2, \bar{s}_0)$ we first determine a_0 and a_1 by satisfying the dispersion sum rules (5) :

$$(4) \quad 2a_0 - 5a_2 - 18a_1 = \frac{16}{\pi}\int_4^\infty ds' \times \mathcal{I}m\left[2F^0 - 5F^2 + 3\left(\frac{4-3s'}{s'-4}\right)F^1\right]\frac{1}{s'^2(s'-4)},$$

$$(5) \quad 2a_0 - 5a_2 = \frac{4}{\pi}\int_4^\infty ds'\, \mathcal{I}m\,(2F^0 + 3F^1 - 5F^2)\frac{1}{s'(s'-4)},$$

where $F^I \equiv F^I(s', t = 0)$ are the forward total amplitudes. In Equation (4) the kernel decreases as S'^{-3} and it would be a good approximation to retain only s- and p-wave contributions. In Equation (5) however, one has to take care of higher waves (f_0 resonance) and asymptotic contributions (Reggeized ρ-exchange). As in Reference [4], we use the Reggeized ρ-parameters given by pole extrapolation with a scale parameter of 0.8 (GeV)^2, and this leads to a total contribution of $l \geq 2$ and asymptotic terms to the RHS of Equation (5) of [4] $\simeq 0.23$. We insist that this is an important ingredient in the present analysis : it is clear that once a_2 is fixed, a_0 depends directly on the value of the asymptotic contributions to Equation (5) for which we have made a choice which seems reasonable but is certainly not compulsory at present.

We then vary the two parameters a_2 and $\bar{s}_0$, keeping the experimental fits for δ_0^0 and δ_0^2 in $\bar{s}_I \leq s \leq 50$ as indicated in (i) above, and retain only the values of a_2 and $\bar{s}_0$ which lead to acceptable agreement between RHS and LHS of Equation (1) in the low-energy region $4 \leq s \leq m_\rho^2$. Owing to the crudeness of our parametrization, we consider satisfactory to have $|\text{LHS} - \text{RHS}|/|\text{RHS}| \leq 10\,\%$ for $s < m_\rho^2$. We may remark that we are not concerned with solving the « up-down » ambiguity for $s > m_\rho^2$ (all our conclusions hold in either case), which would require a local investigation of the equations for $s \geq m_\rho^2$. Also, we notice that subthreshold zeroes (which may become zeroes above threshold for δ_0^2 if $a_2 > 0$) are always present in s-wave amplitudes; this is related directly to the fact that in the experimental region δ_0^0 and δ_0^2 are of opposite signs.

The results can be summarized as follows :

1. If we restrict ourselves to the fit of δ_0^2 as given by Baton *et al.* [1], it appears that the only acceptable solutions require a *positive* $I = 2$ scattering length ($a_2 \sim 0.04 - 0.08$). The corresponding $I = 0$ scattering length is then quite large ($a_0 \sim 0.5$ or 0.6) and not well cons-

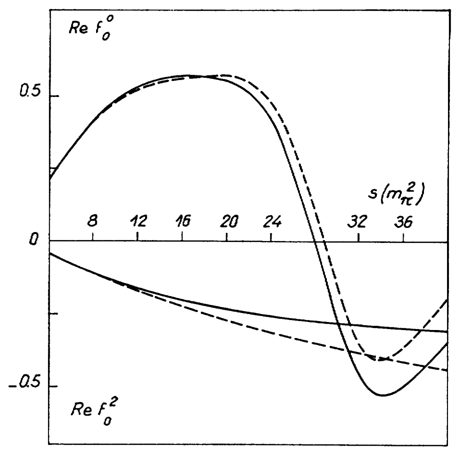

Fig. 2. — Example of solution with $a_2 = -0.04$ and $a_0 = 0.21$. Solid curves: direct calculations; dashed curves : as computed through RHS of Equation (1).

trained. This result for a_0 would be in good agreement with the recent K e^4 data [7] ($a_0 \sim 0.6 \pm 0.25$). However we have a violent disagreement with the generally assumed predictions of Weinberg [8] or of the Lovelace-Veneziano model [6].

2. In analysing the reason for this effect, we found that if δ_0^2 could be somewhat larger in absolute value than the value given by Baton *et al.* [1], things could be restored. Since the $|\delta_0^2|$ of Colton *et al.* [9] is indeed somewhat larger than Baton's, it is possible that some systematic error is present in the latter experiment, and we have allowed for values of δ_0^2 which would lie at the lower end of Baton's error bars. It is then possible to recover agreement with $a_2 < 0$ (see *fig.* 2) and the ranges of the various parameters are [δ_0^2 being for $s \gtrsim 13$ curve (b') of *fig.* 1] :

$$0.18 \leqq a_0 \leqq 0.26, \qquad 0.037 \leqq a_1 \leqq 0.038, \qquad -0.05 \leqq a_2 \leqq -0.03,$$
(« up » case)

$$0.25 \leqq a_0 \leqq 0.29, \qquad 0.040 \leqq a_1 \leqq 0.041, \qquad -0.04 \leqq a_2 \leqq -0.03;$$
(« down » case)

3. To conclude : it appears that fixed-t dispersion relations, in the form of Roy's equations, are a very sensitive and useful tool to investigate the low energy $\pi\pi$ amplitude. We beleive that the correct way to fit (smooth) experimental data is not to fit each wave separately, but all the waves simultaneously with such a dispersion relation analysis. In particular, we think that otherwise any attempt to determine the scattering lengths by extrapolation from the region above threshold is completely arbitrary.

Our present simple minded investigation is perhaps not refined enough to give an unambiguous answer, and a more general global parametrization is certainly necessary, which would have to take into account more carefully contributions from $l \geq 2$ waves and inelasticities together with a use of other possible values for the asymptotic contribution to the sum rule (5) relating the two s-wave scattering lengths.

Our last remark is that in view of our results, it seems that in order to determine unambiguously the low energy $\pi\pi$ S waves one needs more information than the parameters of the ρ-meson and the absence of exotic resonances, as opposed to what was found in other investigations [4] using crossing sum rules in unphysical regions.

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