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Section A :
Physique théorique.

Dynamical formulations of the concept of partial conservation of the axial-vector current

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ABSTRACT. — The concept of Partial Conservation of the Axial-vector Current (PCAC) is analysed in an attempt to extend it to incorporate dynamics. In particular the property of slow variation (smoothness) of the pion amplitudes defined by PCAC in the external four-momentum squared is studied, and it is found that the simplest, mathematically well defined form of smoothness is not possible for a universal dynamical formulation of PCAC.

Some *a priori* new candidates for the interpolating pion field expressed in terms of the basic current operators are also considered, but are found to be equivalent to the PCAC pion field.

Finally we propose a modified formulation of PCAC which does not face the troubles met earlier in the analysis.

1. INTRODUCTION

The current algebra (CA) commutation relations of Gell-Mann [1] can be considered as an equal-time sub-algebra of the algebra of observable field operators for the hadron system. A natural way to exploit this algebra is to calculate its possible representations. Despite much effort very little progress has so far been made in this direction, even for the non-relativistic CA models [2]. At an early stage another philosophy was therefore developed. Instead of looking for the possible representations of the equal-time algebra, one feeds in the partial knowledge of the spectrality of the current operators that one knows from the weak interaction data.

Inside this philosophy the notion of Partial Conservation of the Axial-vector Current operator (PCAC) ([3], [4], [5]) plays a dominant role in that it connects the weak interaction axial-vector current operator $\mathcal{F}_i^{\mu 5}(x)$ $(i = 1, 2, 3;\ \mu = 0, 1, 2, 3)$ to the interpolating pion field operator $\varphi_i(x)$ by the relation

$$\partial_\mu \mathcal{F}_i^{\mu 5}(x) = \text{const.}\ \varphi_i(x). \tag{1.1}$$

Equation (1.1) which can be considered as a defining relation for the pion field operator in view of the works by Haag [6], Nishijima [7] and Zimmermann [8] on the arbitrariness of the interpolating field operators relative to the S-matrix, is further supplemented with a slow variation assumption on the behaviour of the hadron amplitude in the external four-momentum squared of the pions when defined by the field (1.1). The property of slow variation will be called " smoothness " in the following.

The technique developed by Adler [9] to utilize PCAC in hadron physics, the so-called soft-pion technique, later systematized in the form of " effective Lagrangians ", relates the hadronic pion amplitude at zero pion mass and four-momentum, to the weak axial-vector amplitudes. Despite the interesting results obtained by this method, the approach cannot be considered as satisfactory, since it gives rise to knowledge of the amplitude only at one point, thus giving at most a sort of normalization scheme for amplitudes. It therefore seems natural to try to extend and generalize the notion of PCAC to incorporate into the scheme at least some dynamical features. In the present paper we will discuss some of the difficulties met in undertaking this task.

In the first part of the paper we consider the smoothness property, and discuss various extensions of it in order to incorporate dynamics. In section 5 we point out that independently of the definition of the pion field operator, the required smoothness property cannot be universally fulfiled.

In section 6 we then discuss other candidates for the pion field operator than (1.1) and show that inside a big class of models, they are all equivalent to the pion field operator in (1.1). Lastly we show how to modify the smoothness assumption so as to circumvent the troubles met in section 5.

2. THE FIELD THEORETIC BACK-GROUND

We will discuss PCAC inside the general theory of quantized fields. Since the dependence of the amplitudes in the four-momentum variables will be of concern we will mostly work at the level of the LSZ formalism [10], which can be justified by the Haag-Ruelle theory ([6], [11]).

In fact, since we know that

$$(2.1) \qquad \left(\Omega,\, \partial_\mu \mathcal{F}_i^{\mu 5}(x)\, P_{\pi_i}\, \partial_\mu \mathcal{F}_i^{\mu 5}(y)\, \Omega\right) \sim \Delta'(x-y, m_\pi),$$

where P_π is the projection operator on the one pion state, we can construct operators $\Phi_i(t)$ that applied to the vacuum state Ω converges strongly as $t \to \pm\infty$ to the external field operators $\varphi_i^{out}(x)$ and $\varphi_i^{in}(x)$.

In general, however, one must consider equation (1.1) in a stronger sense namely that $\partial_\mu \mathcal{F}_i^{\mu 5}(x)$ and $\varphi_i(x)$ are in the same Borchers class of relatively local S-equivalent field operators. This among other things requires that $\partial_\mu \mathcal{F}_i^{\mu 5}(x)$ and $\varphi_i(x)$ transform under the same representation of the Poincaré-group. To prove that this is actually the case in field theoretical models seems to be a difficult problem and in the following we will assume that this is the case.

It is clear from the types of algorithms used in CA to utilize the divergence of the axial-vector current as an interpolating pion field operator [12] that one cannot project out the pion amplitudes by applying the Klein-Gordon operator on the fields, since the contribution from the current commutator (which does not contain the pion singularities) will then vanish. PCAC therefore also contains an assumption about the behaviour of the matrix-elements of $\partial_\mu \mathcal{F}_i^{\mu 5}(x)$ in the vicinity of the pion mass-shell, and this is what makes it possible to utilize the partial knowledge of the spectrality of $\mathcal{F}_i^{\mu 5}(x)$.

The discussion in the following two sections will be concerned with the problem of how to define the assumption on the variation of the amplitude in the vicinity of the mass-shell.

3. WHAT IS SMOOTHNESS ?

Let us write down the relation obtained by sandwiching the relation

$$(3.1) \qquad \partial_\mu \mathcal{F}_i^{\mu 5}(x) = \frac{1}{\sqrt{2}} f_\pi\, m_\pi^2\, \varphi_i(x)$$

between nucleon states. We have [13] :

$$(3.2) \quad GK(q^2) = \frac{GK(m_\pi^2)\, q^2}{m_\pi^2} - \left(f_\pi\, m_\pi^2 \sqrt{2}\right)^{-1}$$
$$\times \left\{ 2M(q^2 - m_\pi^2)\, g_A(q^2) + (q^2 - m_\pi^2)\, q^2\, \bar{h}_A(q^2) \right\},$$

where $K(q^2)$ is the πN form factor, f_π is the pion decay constant, G is the πN coupling constant, $g_A(q^2)$ the axial-vector form factor, $\bar{h}_A(q^2)$ the non pion pole part of the induced pseudoscalar form factor, m_π is the pion mass and M is the nucleon mass.

The soft pion technique says that we should take $q = 0$, hence $q^2 = 0$. This gives ([3], [4], [5], [14]) :

$$\text{(3.3)} \qquad \mathrm{GK}\,(0) = \frac{\sqrt{2}\,\mathrm{M}\,g_{\mathrm{A}}\,(0)}{f_\pi}$$

which shows that the hadronic matrix-element GK (0) can be calculated at an unphysical piont from the knowledge of $g_{\mathrm{A}}\,(0)$ and f_π. To get contact with physics one invokes smoothness to justify the approximation $\mathrm{GK}\,(0) \approx \mathrm{GK}\,(m_\pi^2) \equiv \mathrm{G}$, the " physical " value.

If we instead expand equation (3.2) around $q^2 = m_\pi^2$ and identify coefficients we get :

$$\text{(3.4 } a\text{)} \qquad \mathrm{GK}\;(m_\pi^2) = \mathrm{GK}\,(m_\pi^2),$$

$$\text{(3.4 } b\text{)} \qquad \mathrm{GK}'\,(m_\pi^2) = \frac{\mathrm{GK}\,(m_\pi^2)}{m_\pi^2} - (f_\pi\, m_\pi^2 \sqrt{2})^{-1} \times \left(2\,\mathrm{M}\,g_{\mathrm{A}}\,(m_\pi^2) + m_\pi^2\,\bar{h}_{\mathrm{A}}\,(m_\pi^2)\right), \qquad \ldots\,.$$

We can now define the smoothness assumption as the requirement

$$\text{(3.5)} \qquad m_\pi^2 \mid \mathrm{K}'\,(m_\pi^2) \mid \ll 1.$$

We then get from (3.4 b) the relation

$$\text{(3.6)} \qquad \mathrm{GK}\,(m_\pi^2) = \frac{2\,\mathrm{M}\,g_{\mathrm{A}}\,(m_\pi^2) + m_\pi^2\,\bar{h}_{\mathrm{A}}\,(m_\pi^2)}{f_\pi \sqrt{2}}.$$

Thus we can calculate the physical hadron matrix-element $\mathrm{GK}\,(m_\pi^2)$ in terms of the weak interaction parameters $g_{\mathrm{A}}\,(m_\pi^2)$, $\bar{h}_{\mathrm{A}}\,(m_\pi^2)$ and f_π. This method clearly generalizes the soft-pion method. It is trivial to remark that one can make the stronger hypothesis of the vanishing (or neglibility) of any number of the higher derivatives. This will have some interest, as we shall see later. In the following we will discuss the nature of the smoothness assumption (3.5).

We first notice that

$$\mathrm{K}'\,(m_\pi^2) = \frac{d\mathrm{K}}{dq^2}\,(m_\pi^2) \simeq \frac{1}{6} \langle\, r^2 \,\rangle_{\pi\,\mathrm{N}},$$

where $\langle\, r^2 \,\rangle_{\pi\,\mathrm{N}}$ is the mean-square radius of the hadronic charge distribution of the nucleon. To assume that this is negligibly small means essentially that the pions are coupled to the nucleons as if the latter were piont particles. As we shall see later, this indicates that the nature of the smoothness assumption is to linearize the pion interaction. In the soft pion approach this is expressed in the tree approximation, which replaces the smoothness assumption in the algorithms for effective Lagrangians.

The extension of the smoothness assumption discussed above to more complicated processes was done in reference [13], but in order not to obscure the nature of the PCAC assumption by CA we shall illustrate below the method with a different example.

4. UNIFORM SMOOTHNESS

Consider the matrix-element

$$(4.1)\quad \langle P'; q, j \text{ out} \,|\, \mathcal{F}_i^{\mu 5}(0) \,|\, P \rangle$$
$$= \bar{u}(P')\left\{ \delta_{ij}\, F^{\mu(+)}(k^2, s, t) + \frac{1}{2}[\tau_i, \tau_j]\, F^{\mu(-)}(k^2, s, t) \right\} u(P)$$

describing axial-vector production of pions by nucleons.

The kinematics is given by $s = (P + k)^2$ and $t = (P' - P)^2$, k is the four-momentum of the axial-vector current, q that of the pion with isospin index j, and the τ_i:s are the Pauli-matrices describing the isospin properties of the amplitude.

By splitting off the pion pole at $k^2 = m_\pi^2$ we get

$$(4.2)\qquad F^{\mu(\pm)}(k^2, s, t) = \frac{ik^\mu f_\pi T^{(\pm)}(m_\pi^2, s, t)}{(k^2 - m_\pi^2)\sqrt{2}} - i\, G^{\mu(\pm)}(k^2, s, t),$$

where

$$T^{(\pm)}(m^2, s, t) = A^{(\pm)}(s, t) + \frac{1}{2}(\mathbf{k} + \mathbf{q})\, B^{(\pm)}(s, t)$$

are the (on mass-shell) pion nucleon scattering amplitudes.

By taking the divergence of equation (4.1) and using (3.1) as well as (4.2) we get

$$(4.3)\qquad m_\pi^2\, c_\pi\, T^{(\pm)}(k^2, s, t) = k^2\, c_\pi\, T^{(\pm)}(m_\pi^2, s, t) - (k^2 - m_\pi^2)\, k_\mu\, G^{\mu(\pm)}(k^2, s, t),$$

where we have introduced the abbreviation $c_\pi = \dfrac{f_\pi}{\sqrt{2}}$. Expanding both sides around $k^2 = m_\pi^2$ and identifying coefficients gives

$$(4.4\,a)\qquad m_\pi^2\, c_\pi\, T^{(\pm)}(m_\pi^2, s, t) = m_\pi^2\, c_\pi\, T^{(\pm)}(m_\pi^2, s, t),$$

$$(4.4\,b)\qquad m_\pi^2\, c_\pi \frac{d}{dk^2} T^{(\pm)}(k^2, s, t)\Big|_{k^2 = m_\pi^2} = c_\pi\, T^{(\pm)}(m_\pi^2, s, t) - k_\mu\, G^{\mu(\pm)}(m_\pi^2, s, t), \qquad \ldots$$

Equation (4.4 a) is just a verification of the HNZ theorem ([6], [7], [8]), and shows that the pion field has been correctly normalized. The smoothness assumption which is relevant to this case is the following one :

$$(4.5) \qquad m_\pi^2 \left| \frac{d}{dk^2} \mathrm{T}^{(\pm)}(k^2, s, t) \right|_{k^2 = m_\pi^2} \ll | \mathrm{T}^{(\pm)}(m_\pi^2, s, t) |$$

uniformly in s for fixed t, or in other words that the left hand side of equation (4.4 b) can be neglected for all s and fixed t. We then get the relation

$$(4.6) \qquad c_\pi \mathrm{T}^{(\pm)}(m_\pi^2, s, t) = k_\mu \mathrm{G}^{\mu(\pm)}(m_\pi^2, s, t).$$

This relation thus holds for all s with t fixed.

G^μ is decomposed into invariants as follows [15] :

$$(4.7) \quad \mathrm{G}^\mu(m_\pi^2, s, t) = \mathrm{F}_1 \frac{1}{2}[\mathbf{q}, \gamma^\mu] + \mathrm{F}_2(\mathrm{P}' + \mathrm{P})^\mu + \mathrm{F}_3 q^\mu + \mathrm{F}_4 \mathrm{M} \gamma^\mu + \mathrm{F}_5 \mathbf{k}(\mathrm{P} + \mathrm{P}')^\mu + \mathrm{F}_6 \mathbf{k} q^\mu + \mathrm{F}_7 k^\mu + \mathrm{F}_8 \mathbf{k} k^\mu.$$

Here $\mathrm{F}_i = \mathrm{F}_i(m_\pi^2, s, t)$ and the expansion holds for both the isospin odd and even amplitudes. Insertion of (4.7) into (4.6) gives

$$\begin{aligned}
(4.8\,a) \quad c_\pi \mathrm{A}^{(\pm)}(s, t) = {} & \mathrm{F}_1^{(\pm)}(m_\pi^2, s, t)(k\mathrm{P} + k\mathrm{P}') \\
& + \mathrm{F}_2^{(\pm)}(m_\pi^2, s, t)(k\mathrm{P} + k\mathrm{P}') \\
& + \mathrm{F}_3^{(\pm)}(m_\pi^2, s, t)\, kq + \mathrm{F}_7^{(\pm)}(m_\pi^2, s, t)\, m_\pi^2, \\
(4.8\,b) \quad c_\pi \mathrm{B}^{(\pm)}(s, t) = {} & - \mathrm{F}_1^{(\pm)}(m_\pi^2, s, t)\, 2\mathrm{M} + \mathrm{F}_4^{(\pm)}(m_\pi^2, s, t)\, \mathrm{M} \\
& + \mathrm{F}_5^{(\pm)}(m_\pi^2, s, t)(k\mathrm{P} + k\mathrm{P}') \\
& + \mathrm{F}_6^{(\pm)}(m_\pi^2, s, t)\, kq + \mathrm{F}_8^{(\pm)}(m_\pi^2, s, t)\, m_\pi^2.
\end{aligned}$$

This general relation expresses the π N amplitudes $\mathrm{A}^{(\pm)}$ and $\mathrm{B}^{(\pm)}$ in terms of the weak interaction amplitudes for any s and fixed t.

At this piont it should be noticed, that in contrast, the soft-pion method in this case is completely ambigous, since it is not clear to which physical piont one should continue the corresponding soft-pion result for the π N amplitudes. In our case there is a precise statement on the nature of the smoothness assumption and no ambiguity occurs.

The relations (4.8 a) and (4.8 b) can be used in several ways.

If we calculate the Born-terms explicitly it is easy to see that

$$\begin{aligned}
\mathrm{T}^{(+)}((m_\pi + \mathrm{M})^2, 0) = {} & \mathrm{A}^{(+)}((m_\pi + \mathrm{M})^2, 0) \\
& + m_\pi \mathrm{B}^{(+)}((m_\pi + \mathrm{M})^2, 0) = \mathcal{O}(m_\pi^2)
\end{aligned}$$

which is the content of Adler's consistency condition [9] here at the physical threshold and on the pion mass-shell.

One should, however, also remember that the relations (4.8 *a*) and (4.8 *b*) go in both directions. Thus they can be used to get information about the behaviour of the weak amplitudes as well. As an example let us parametrize the high-energy behaviour of the amplitudes $A^{(-)}$ and $B^{(-)}$ by the Regge-pole model. We have

$$\left.\begin{array}{l} A^{(-)}(\nu, 0) \sim \nu^{\alpha_\rho(0)} \\ B^{(-)}(\nu, 0) \sim \nu^{\alpha_\rho(0)-1} \end{array}\right\} \text{as} \quad \nu \to \infty,$$

where $\nu = \frac{1}{2\,M}(S - M^2 - m_\pi^2)$ and $\alpha_\rho(0)$ is the intercept at $t = 0$ of the ρ-trajectory. From this follows that when $\nu \to \infty$ we have

$$\begin{array}{ll} F_i^{(-)}(m_\pi^2, \nu, 0) \sim \nu^{\alpha_\rho(0)} & \text{for} \quad i = 3, 7, \\ F_i^{(-)}(m_\pi^2, \nu, 0) \sim \nu^{\alpha_\rho(0)-1} & \text{for} \quad i = 1, 2, 4, 6, 8. \\ F_5^{(-)}(m_\pi^2, \nu, 0) \sim \nu^{\alpha_\rho(0)-2} & \end{array}$$

The experimental value of $\alpha_\rho(0)$ is $\alpha_\rho(0) \approx 0.5$.

The behaviour of the amplitude $F_5^{(-)}(m_\pi^2, \nu, 0)$ as $\nu \to \infty$ together with the crossing relation

$$F_5^{(-)}(m^2, -\nu, 0) = -F_5^{(-)}(m^2, \nu, 0)$$

then leads to the superconvergence relation [16] :

$$\int_{m_\pi}^{\infty} d\nu \, \mathrm{Im}\, F_5^{(-)}(m_\pi^2, \nu, 0) = 0. \tag{4.9}$$

This relation is an independent test of uniform smoothness in the high-energy region.

5. PROBLEMS WITH SMOOTHNESS

It is tempting to try to sharpen the uniform smoothness assumption used in section 4 above to

$$\frac{d}{dk^2} T^{(\pm)}(k^2, s, t)\Big|_{k^2 = m_\pi^2} = 0 \tag{5.1}$$

and in the general case to require that all pion amplitudes should satisfy similar relations, what we would call " exact uniform smoothness ". Such a requirement does not seem to clash in any obvious way with the S-matrix formalism in the LSZ [10] scheme, since the physical S-matrix elements do not depend on the value of the derivatives of the amplitudes.

Unfortunately this scheme does not work, as can be seen from a study of the pion electromagnetic form factor [17]. This is of course strictly speaking not an S-matrix element, but in a theory with currents it seems resonable that one also should be able to consider such matrix-elements.

Let us define the off mass-shell form factor by the LSZ formula

$$(5.2)\quad \begin{aligned} \mathrm{M}^{\mu}(p'^2, p^2, t) &= i^2(-p'^2+m_\pi^2)(-p^2+m_\pi^2)\int d^4x\, d^4y\, e^{ip'x-py} \\ &\quad \times(\Omega, \mathrm{T}(\varphi(x)\,\mathrm{J}^{\mu}(0)\,\varphi^+(y))\,\Omega) \\ &= (p'+p)^{\mu}\,\mathrm{F}_1(p'^2, p^2, t) + (p'-p)^{\mu}\,\mathrm{F}_2(p'^2, p^2, t), \end{aligned}$$

where $t = (p'-p)^2$.

Now

$$(5.3)\quad (p'-p)_{\mu}\,\mathrm{M}^{\mu}(p'^2, p^2, t) = (p'^2-p^2)\,\mathrm{F}_1(p'^2, p^2, t) + t\,\mathrm{F}_2(p'^2, p^2, t).$$

On the other hand, using the commutation relations

$$(5.4\,a)\quad \delta(x^0)\,[\mathrm{J}^0(0), \varphi(x)] = \delta^4(x)\,\varphi(x),$$

$$(5.4\,b)\quad \delta(x^0)\,[\mathrm{J}^0(0), \varphi^+(x)] = -\,\delta^4(x)\,\varphi^+(x)$$

gives the Ward-Takahashi identity

$$(5.5)\quad \begin{aligned} (-p'^2+m_\pi^2)\,d_\pi(p^2) - (-p^2+m_\pi^2)\,d_\pi(p'^2) \\ = (p'-p)_{\mu}\,\mathrm{M}^{\mu}(p'^2, p^2, t), \end{aligned}$$

where

$$d_\pi(p^2) = (-p^2+m_\pi^2)\,\Delta'_{\mathrm{F}}(p^2).$$

$\Delta'_{\mathrm{F}}(p^2)$ is the time ordered two-piont function of the pion field operator in momentum space.

If we put $p'^2 = m_\pi^2$ and differentiate both sides of (5.5) with respect to p^2 at $p^2 = m_\pi^2$ we get :

$$(5.6)\quad \mathrm{F}_1(m_\pi^2, m_\pi^2, t) - t\left(\frac{d}{dp^2}\mathrm{F}_2\right)(m_\pi^2, m_\pi^2, t) = 1.$$

Hence if $\frac{d\mathrm{F}_2}{dp^2}(m_\pi^2, m_\pi^2, t) = 0$ for all t we get

$$(5.7)\quad \mathrm{F}_1(m_\pi^2, m_\pi^2, t) = 1 = \text{constant}.$$

This is clearly unacceptable, and excludes essentially any π-π interaction in the isospin 1 channel. This was anticipated in section 4. Unfortunately uniform smoothness does not work either in this case since it requires

$$(5.8)\quad \left|m_\pi^2\frac{d\mathrm{F}_2}{dp^2}(m_\pi^2, m_\pi^2, t)\right| \ll |\,\mathrm{F}_2(m_\pi^2, m_\pi^2, t)\,|.$$

But since $F_2(m_\pi^2, m_\pi^2, t) = 0$ from current conservation this gives again equation (5.7).

The absence of the π-π interactions shows that PCAC has the effect of linearizing the pion interactions. Hence one should expect troubles with PCAC in the sector of Hilbert space with baryonic number $B = 0$. For $B \neq 0$, however, this approximation can still be valuable, corresponding to the idea that the mesonic effects are of order $\frac{m_\pi}{M}$ where M is some baryonic mass, compared to the direct coupling. This makes sense in the sectors $B \neq 0$ but not in the sector $B = 0$.

6. MODIFICATIONS OF PCAC

In view of the difficulties discussed above it is clear that we must modify PCAC if we want to incorporate dynamics into it. Especially, we must find another definition of smoothness, since the troubles in section 5 are independent of how we define the pion field. We start, however, by investigating some alternative possibilities to define the pion field operator in terms of current operators.

A. Smoother pion fields

Since we are concerned with the properties of the pion field off the mass-shell it is natural to study the sub-class $\mathcal{B}' \subset \mathcal{B}$ of the Borchers' class [18] $\mathcal{B}$ of S-equivalent local, relatively local pion field operators given by

$$(6.1) \qquad \mathcal{B}' = \{ \varphi_i(x);\ \varphi_i(x) = \mathcal{P}_N(\mathcal{K}_{m_\pi^2})\, \partial_\mu \mathcal{F}_i^{\mu 5}(x),\ \forall\ N \in Z_+ \},$$

where $\mathcal{P}_N(a)$ is a polynomial of degree $N \geq 0$, and $\mathcal{K}_{m_\pi^2}$ is the Klein-Gordon operator.

Our form of smoothness as developed in sections 4 and 5 gives rise to the normalization of two coefficients in $\mathcal{P}_N(a)$. Let

$$\mathcal{P}_N(a) = A_{N,0} + A_{N,1}\, a + \ldots + A_{N,N}\, a^N;$$

then the mass-shell value of the S-matrix-elements gives

$$(6.2) \qquad A_{N,0} = \frac{\sqrt{2}}{f_\pi\, m_\pi^2}, \quad \forall\ N$$

as in (3.1). The π N vertex allows us to take

$$(6.3) \qquad A_{N,1} = G - \frac{2\, M\, g_A(m_\pi^2) + m_\pi^2\, \bar{h}_A(m_\pi^2)}{\sqrt{2}\, f_\pi\, m_\pi^2}.$$

Thus for the π N vertex the smoothness property is exactly fulfiled by construction.

B. Composite pion field operators

Following Nishijima [7] and Zimmermann [8] we can choose for $\varphi_i(x)$ any local object that has the correct transformation properties. Thus we may consider pion fields of the form $[\mathcal{F}^{\mu}_{k}(x)$ is the isospin current$]$:

$$(6.4)\quad \varphi_i(x) = \lim_{\substack{\xi\to 0\\ \xi^2<0}} \frac{\varepsilon_{ijk}\,\mathrm{T}\left(\mathcal{F}^{\mu 5}_{j}\left(x-\frac{1}{2}\xi\right)\mathcal{F}_{k\mu}\left(x+\frac{1}{2}\xi\right)\right)}{\left(\Omega,\ \varepsilon_{ijk}\,\mathrm{T}\left(\mathcal{F}^{\mu 5}_{j}\left(x-\frac{1}{2}\xi\right)\mathcal{F}_{k\mu}\left(x+\frac{1}{2}\xi\right)\right)\Psi_{\pi_i}(p)\right)}$$

or any other combination of internal quantum-numbers that gives rise to correct isospin and space reflection properties. For some combinations of strangeness changing currents (6.4) might however not exist.

The construction of the pion field above gives an operator that is local and relatively local to the currents. One would therefore expect it to be in the Borchers' class of $\partial_\mu\,\mathcal{F}^{\mu 5}_{i}(x)$. Indeed if the leading singularity of (6.4) is given by the Bjorken expansion [19] it is easy to check that both in the U (6)$\times$U (6) quark-model current algebra and in the algebra of fields model, the expression (6.4) is exactly equivalent to $\partial_\mu\,\mathcal{F}^{\mu 5}_{i}(x)$.

The conditions for not getting a new field has been investigated inside perturbation theory by Nishijima ([20], [21]) . He finds that a new field is not introduced if

(1) The self-energy diverges;

(2) The vertex function is convergent or less divergent than the self-energy.

These conditions are not immediately translatable to non perturbative field theory but from the experience with models the conjecture is that for strong interactions the construction above does not give a new field, or at least not a field outside the type of subclass of the Borchers' class of the original field described in A.

C. Modified PCAC

Having seen above that the choice of pion field is rather limited inside the philosophy sketched in the introduction we want to show how one can modify both definition (1.1) and the smoothness condition to circumvent the problem of the pion electromagnetic form factor.

We define the interpolating pion field by the formula

$$\varphi_i(x) = -\frac{\sqrt{2}}{f_\pi\, m_\pi^4}\, \Box\, \partial_\mu\, \mathcal{F}_i^{\mu 5}(x). \tag{6.5}$$

This is clearly a special case of $\mathcal{B}'$ with

$$A_{1,0} = c_\pi^{-1}\, m_\pi^{-2} \qquad \text{and} \qquad A_{1,1} = -\, c_\pi^{-1}\, m_\pi^{-4}.$$

With this pion field we can take smoothness to mean that *the second order derivative of the amplitude with respect to the external four-momentum squared of the pion is negligibly small.* In the case of the pion electromagnetic form factor we get

$$\frac{dF_1}{dp^2}(p^2, m_\pi^2, t)\bigg|_{p^2 = m_\pi^2} = 0 \tag{6.6}$$

which does not seem to contradict physics.

For the π N form factor we now get by putting $\frac{d^2 K}{d\,(p^2)^2}(m_\pi^2) = 0$:

$$\begin{aligned} GK(m_\pi^2) = {} & \frac{2\,M\, g_A(m_\pi^2) + m_\pi^2\, \bar{h}_A(m_\pi^2)}{\sqrt{2}\, f_\pi} \\ & + \frac{2\,M\, g'_A(m_\pi^2) + m_\pi^2\, \bar{h}'_A(m_\pi^2) + \bar{h}_A(m_\pi^2)}{\sqrt{2}\, f_\pi}\, m_\pi^2. \end{aligned} \tag{6.7}$$

Since the derivatives of the weak amplitudes enter in this expression, it shows that the price we have to pay is that we must have a more detailed knowledge of the spectrality of the currents.

The extension of the above formalism to other amplitudes and to the case of uniform smoothness described in section 4 is straight forward. The corresponding expressions to (4.8 *a*) and (4.8 *b*) for $A^{(\pm)}$ and $B^{(\pm)}$ will now involve derivatives of the amplitudes $F_i^{(\pm)}$.

As before we can also in this case use a particular member of $\mathcal{B}'$ to let the new smoothness assumption be satisfied by construction for the pion-nucleon vertex. The ultimate test of the above scheme must of course come from experiment when we can reliably determine the derivatives of the matrix-elements of the currents.

7. DISCUSSION AND CONCLUSIONS

Our analysis of PCAC has its origin in a critisism of the soft pion method, as giving no dynamical information of the hadron physics, although the input is in principle a complete knowledge of the weak interaction matrix-elements.

We have in particular concentrated upon the question of how to formulate the smoothness assumption in PCAC, which is introduced in order to utilize the full knowledge of the spectrality of the weak axial-vector current. The discussion in sections 3 and 4 is based upon the idea that as little as possible should be assumed about the off mass-shell behaviour of the pion amplitudes. Thus it is natural to consider the derivatives of the amplitudes at the pion mass-shell. The most economical (from the piont of view of the input) assumption about the smallness of the first order derivative with respect to the external four-momentum squared of the pion amplitude was extended in section 4 to " uniform smoothness ". This concept gives rise to powerful dynamical relations between weak and strong interaction amplitudes. However, in section 5 we found that this form of the smoothness assumption runs into troubles with the dynamics of the pion electromagnetic form factor, and that the smoothness condition essentially linearizes the pion interactions. Although this might still be a good approximation in the sectors of Hilbert space where the baryon number is different from zero, it clearly asks for a modification of PCAC.

After having shown that the possible candidates for the pion field inside a theory of currents are essentially limited to a particular subclass of the Borchers' class of $\partial_\mu \mathcal{F}_i^{\mu 5}(x)$ we formulated in section 6 C a new smoothness condition, which circumvented the troubles with the pion electromagnetic form factor. This new formulation of PCAC, however, requires a more detailed knowledge of the matrix-elements of the weak axial-vector current and is thus correspondingly less useful at present.

In conclusion we therefore find, that the idea to universally connect strong and weak interaction amplitudes in a dynamical way *via* PCAC is, although not in principle impossible (in the form described in section 6 C), however, at present not practically feasible. For restricted cases application of the stronger form of uniform smoothness described in section 4 might still be useful.

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