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Pure radiation fields in general relativity


<http://www.numdam.org/item?id=AIHPA_1967__7_3_245_0>
A METHOD FOR GENERATING
PURE RADIATION FIELDS
FROM EMPTY GRAVITATIONAL FIELDS
IS GIVEN

The object of this investigation is to obtain a method for generating pure radiation fields from empty gravitational fields. Such methods were obtained earlier by Ehlers [1], Bonnor [2] and the present author [3] in other cases.

For this purpose we consider the three metrics

\[ ds^2 = e^{2\sigma} d^* s^2 - \bar{e}^{2\sigma} (dx^3)^2, \]

\[ ds^2 = e^{2W} d^* s^2 - \bar{e}^{2W} (dx^3)^2, \]

\[ d^* s^2 = g_{ij} dx^i dx^j \]

\((i, j, \ldots = 1, 2, 4; \alpha, \beta, \ldots = 1, 2, 3, 4)\)

where \(\sigma, W\) and \(g_{ij}\) are functions of \(x^i\) only.

Hereafter quantities defined with respect to (2) and (3) will be denoted by an overhead bar or an asterisk respectively.

The Ricci tensor for the metric (1) is given by

\[ R_{ij} = R_{ij} + 2\sigma_i \sigma_j + \bar{g}_{ij} \Delta_2 \sigma \]

\[ R_{33} = \bar{e}^{4\sigma} \Delta_2 \sigma, \]

\[ R_{3i} = 0, \]
where

\( \sigma_i = \frac{\partial \sigma}{\partial x^i} \),

and \( *\Delta_2 \) is the Beltrami differential parameter of the second kind.

The empty space-time field equations for the metric (1) are therefore

\[
\begin{align*}
\ \ (8) & \quad *R_{ij} + 2\sigma_i\sigma_j = 0, \\
\ \ (9) & \quad *\Delta_2 \sigma = 0,
\end{align*}
\]

and since (9) follows from (8) in view of the contracted Bianchi identities, we are left with only equations (8) for determining \( \sigma \) and \( *g_{ij} \).

Now the Ricci tensor for the metric (2) is given by

\[
\begin{align*}
\ \ (10) & \quad \bar{R}_{ij} = *R_{ij} + 2W_iW_j + *g_{ij}*\Delta_2 W, \\
\ \ (11) & \quad \bar{R}_{33} = e^{4w}*\Delta_2 W, \\
\ \ (12) & \quad \bar{R}_{3i} = 0,
\end{align*}
\]

where

\[
W_i = \frac{\partial W}{\partial x_i}.
\]

If we assume that \( W \) is a function of \( \sigma \) so that the level surfaces of \( W \) and \( \sigma \) coincide, then

\[
\begin{align*}
\ \ (14) & \quad W_i = W^'\sigma_i, \quad W_{i;j} = W^'\sigma_{i;j} + W^{''}\sigma_i\sigma_j, \ldots
\end{align*}
\]

where an overhead dash denotes ordinary differentiation with respect to \( \sigma \) and a semicolon followed by a lower index denotes covariant differentiation with respect to the metric (3).

From equations (11) and (14) we get

\[
\begin{align*}
\ \ (15) & \quad \bar{R}_{33} = e^{4w}W^*g^{ij}\sigma_i\sigma_j,
\end{align*}
\]

in view of (9). And if we assume that \( \sigma_i \) is a null vector then

\[
\begin{align*}
\ \ (16) & \quad \bar{R}_{33} = 0.
\end{align*}
\]

Also from (10), (14), (8) and (9) and the fact that \( \sigma_i \) is a null vector we get

\[
\begin{align*}
\ \ (17) & \quad \bar{R}_{ij} = 2(W'^2 - 1)\sigma_i\sigma_j.
\end{align*}
\]

Thus the Ricci tensor for the metric (2), in view of (12), (16) and (17), can be written as

\[
\begin{align*}
\ \ (18) & \quad \bar{R}_{\alpha\beta} = \theta_{\alpha\sigma}\sigma_{\beta},
\end{align*}
\]
where

\[ \theta = 2(W^1 - 1), \]

and \( \sigma \beta \) is a null vector.

Now equations (18) are precisely the field equations for the unidirectional flow of pure radiation for the metric (2). Hence we have arrived at the following result:

For every solution of the empty space-time field equations corresponding to the metric (1) a solution of the field equations for the unidirectional flow of pure radiation, i.e. (18), is given by the metric (2) where \( W \) is an arbitrary function of \( \sigma \) and the gradient of \( \sigma \) is null vector.

The author wishes to thank Prof. P. C. Vaidya for helpful discussions.

REFERENCES


(Manuscrit reçu le 14 avril 1967).