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# A COMPACTIFICATION OF $(\mathbb{C}^*)^4$ WITH NO NON-CONSTANT MEROMORPHIC FUNCTIONS

by J.-M. HWANG<sup>1</sup> and D. VAROLIN

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## 1. Introduction.

There is a well-known example, due to J.-P. Serre, of a Zariski open subset of a ruled surface over an elliptic curve, which is Stein, but not affine ([Ha] 6.3). This example plays an interesting role in complex analysis, for example in the theory of local cohomology of analytic sheaves (e.g. [KP]) and the theory of nef vector bundles ([DPS] 1.7). The purpose of this note is to extend this construction to the dimension 4 by interpreting it from the view-point of additive group action. Of course, it may be possible to have more direct generalization of Serre's construction to higher dimensions. But our approach via additive group action reveals a number of interesting features of the resulting 4-dimensional compact complex manifold. This complex manifold is interesting in the following aspects.

A well-known conjecture in the study of compactifications of  $\mathbb{C}^n$  is the following:

CONJECTURE. — *Every compactification of  $\mathbb{C}^n$  is Moishezon. Namely, a compact complex manifold containing  $\mathbb{C}^n$  as a Zariski open subset has  $n$  algebraically independent meromorphic functions.*

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Although this is true in dimension 2, it is completely open in higher-dimensions, except for some partial results in dimension 3 (cf. [PS]). Even under the additional assumption that the compactifying divisor is smooth, in which case it is conjectured that the compactification is  $\mathbb{P}_n$ , or under the assumption that the compactification is Kähler, the problem remains unsolved. One may ask the same question for compactifications of  $(\mathbb{C}^*)^n$ . But our construction will give a negative answer:

**THEOREM 1.** — *Let  $T$  be any complex torus of dimension 2. Then there exists a compact Kähler 4-fold  $X = X(T)$  and a smooth divisor  $D \subset X$  such that  $D$  is biholomorphic to  $\mathbb{P}_1 \times T$  and  $X - D$  is biholomorphic to  $(\mathbb{C}^*)^4$ .*

Since the image of a Moishezon manifold is Moishezon, if  $T$  is not an abelian variety,  $X(T)$  is a compactification of  $(\mathbb{C}^*)^4$  which is not Moishezon.

One partial answer to the above conjecture is the result of Gellhaus [Ge] that any equivariant compactification of  $\mathbb{C}^n$  is Moishezon. In other words, if  $\mathbb{C}^n$  acts on an  $n$ -dimensional compact complex manifold with a faithful orbit, the manifold has  $n$  algebraically independent meromorphic functions. One may ask the following question as a generalization of this result:

*If  $\mathbb{C}^n$  acts on an  $m$ -dimensional compact complex manifold,  $m \geq n$ , with a faithful orbit, does the manifold have at least  $n$  algebraically independent meromorphic functions?*

In fact, answering question of this type is believed to be one of the possible approaches to the above conjecture. However, our manifold  $X(T)$  gives a negative answer again. There is a  $\mathbb{C}^2$ -action on  $X(T)$  with faithful orbits by construction, but

**THEOREM 2.** — *For a general torus  $T$ , the manifold  $X(T)$  has no non-constant meromorphic functions.*

Our example suggests that construction of meromorphic functions on the compactification of  $\mathbb{C}^n$  may be a very delicate problem.

One of the advantage of the view-point of additive group action in our construction is that it can be easily generalized to other cases. In principle, when there is a quotient  $Y'$  of a complex manifold  $Y$  by a lattice in  $\mathbb{C}^k$  we can get a  $\mathbb{C}^k$ -action on  $(\mathbb{C}^*)^k \times Y$  and a compactification of  $(\mathbb{C}^*)^k \times Y$  which is a  $\mathbb{P}_k$ -bundle over  $Y'$ . For example, it is straight-forward to generalize our

construction to a compactification of  $(\mathbb{C}^*)^{2n}$ , which is a  $\mathbb{P}_n$ -bundle over an  $n$ -dimensional complex torus.

## 2. An approach to Serre's example via $\mathbb{C}$ -action.

It is instructive first to give a construction of Serre's example from the view-point of  $\mathbb{C}$ -action on  $\mathbb{C}^* \times \mathbb{C}^*$ , to clarify the construction in Theorem 1.

Let  $\alpha \in \mathbb{C} - \mathbb{R}$ . Consider the  $\mathbb{C}$ -action on  $\mathbb{C}^* \times \mathbb{C}^*$  given by

$$(1) \quad s \cdot (x, z) = (e^s x, e^{\alpha s} z).$$

Since  $\alpha$  and 1 are independent over  $\mathbb{Z}$ , the map  $s \mapsto s \cdot p$  is injective for any  $p \in \mathbb{C}^* \times \mathbb{C}^*$ , i.e., the action is faithful. Moreover, since  $\alpha$  and 1 are independent over  $\mathbb{R}$  the same map is actually proper. Indeed, if  $\{s_j\}_{j \in \mathbb{N}}$  is a divergent sequence in  $\mathbb{C}$  such that  $\{\log |e^{s_j} x|\}_{j \in \mathbb{N}}$  is bounded, then the real part of  $s_j$  is confined to a strip of finite width in the  $s$ -plane for all  $j$ . Thus the imaginary part of  $s_j$  diverges. But since  $\alpha$  has non-zero imaginary part,  $\log |e^{\alpha s} z|$  is unbounded.

It follows from general theory of Lie group actions that the quotient of  $\mathbb{C}^* \times \mathbb{C}^*$  by the action (1) is a Riemann surface  $B$ . We claim in fact that it is an elliptic curve. Indeed, this action realizes  $\mathbb{C}^* \times \mathbb{C}^*$  as a locally trivial  $\mathbb{C}$ -bundle over  $B$ , and thus in particular,  $B$  is homotopy equivalent to  $\mathbb{C}^* \times \mathbb{C}^*$ . Since every noncompact Riemann surface has no second homology, we see that  $B$  must be compact. The homology of  $B$  then forces it to be an elliptic curve. In fact, one can check that  $B$  is the torus  $\mathbb{C}/(\mathbb{Z} + \alpha\mathbb{Z})$ .

Now, since  $\text{Aut}(\mathbb{C})$  is an affine group, the bundle  $\mathbb{C}^* \times \mathbb{C}^* \rightarrow B$  is an affine bundle with fibers  $\mathbb{C}$ . Thus we can attach  $\infty$  to each fiber and obtain a  $\mathbb{P}_1$ -bundle over the elliptic curve  $B$ . Equivalently, the affine transition functions of the bundle  $\mathbb{C}^* \times \mathbb{C}^* \rightarrow B$  can be homogenized so as to define a rank 2 vector bundle  $E \rightarrow B$  whose projectivization  $\mathbb{P}(E) \rightarrow B$  has a distinguished section, and the complement of this section is  $\mathbb{C}^* \times \mathbb{C}^*$ .

The construction outlined above can be carried out quite explicitly, and the reader is invited to do so and obtain in particular the following additional facts:

- The bundle  $\mathbb{P}(E) \rightarrow B$  is real analytically isomorphic to  $\mathbb{P}_1 \times B$ . Thus the section of this bundle has self intersection 0. However, since the complement of this section is Stein, the section is holomorphically rigid.

- The vector bundle  $E \rightarrow B$  is flat, i.e., it can be given transition functions which are locally constant.
- In fact,  $E \rightarrow B$  is a non-split extension of  $\mathcal{O}$  by  $\mathcal{O}$ , and thus the algebraic structure inherited by  $\mathbb{C}^* \times \mathbb{C}^*$  from  $\mathbb{P}(E)$  is not affine.

*Remark.* — A famous problem in complex analysis is to determine whether or not  $\mathbb{C}^* \times \mathbb{C}^*$  contains an open subset biholomorphic to  $\mathbb{C}^2$  ([RR] Appendix). Perhaps one can show that no open subset of  $\mathbb{P}(E)$  is biholomorphic to  $\mathbb{C}^2$ .

### 3. Proof of Theorem 1.

Every complex torus  $T$  is biholomorphic to one of the form  $\mathbb{C}^2 / (\mathbb{Z}e_1 + \mathbb{Z}e_2 + \mathbb{Z}\lambda + \mathbb{Z}\mu)$ , where  $e_1, e_2$  are the standard unit vectors in  $\mathbb{C}^2$  and  $\{e_1, e_2, \lambda, \mu\}$  are independent over  $\mathbb{R}$ . We call  $\{\lambda, \mu\}$  *normalized lattice vectors* for  $T$ . In terms of normalized lattice vectors  $\lambda = (\lambda^1, \lambda^2)$  and  $\mu = (\mu^1, \mu^2)$ ,  $T$  can be obtained as a quotient of  $\mathbb{C}^* \times \mathbb{C}^*$  by the  $\mathbb{Z}^2$  action

$$(m, n) \cdot (z, w) = (ze^{2\pi\sqrt{-1}(m\lambda^1+n\mu^1)}, we^{2\pi\sqrt{-1}(m\lambda^2+n\mu^2)}).$$

We denote the quotient map by  $(z, w) \mapsto [z, w]$ .

Fix a torus  $T = \mathbb{C}^2/\Gamma = \mathbb{C}^2/(\mathbb{Z}e_1 + \mathbb{Z}e_2 + \mathbb{Z}\lambda + \mathbb{Z}\mu)$  and consider the following  $\mathbb{C}^2$  action on  $(\mathbb{C}^*)^4$ :

$$(2) \quad (s, t) * (x, y, z, w) := (e^s x, e^t y, e^{\lambda^1 s + \mu^1 t} z, e^{\lambda^2 s + \mu^2 t} w),$$

where  $\lambda = \lambda^1 e_1 + \lambda^2 e_2$ , and similarly for  $\mu$ .

First, notice that this is a faithful action. Indeed, for fixed  $p \in (\mathbb{C}^*)^4$ , if  $(s, t) * p = (s', t') * p$  then

$$\begin{aligned} s - s' &= 2\pi\sqrt{-1}m_1 \\ t - t' &= 2\pi\sqrt{-1}m_2 \\ (\lambda^1 s + \mu^1 t) - (\lambda^1 s' + \mu^1 t') &= 2\pi\sqrt{-1}k_1 \\ (\lambda^2 s + \mu^2 t) - (\lambda^2 s' + \mu^2 t') &= 2\pi\sqrt{-1}k_2 \end{aligned}$$

for some integers  $m_1, m_2, k_1, k_2$ . Thus  $m_1\lambda + m_2\mu - k_1e_1 - k_2e_2 = 0$  and so  $m_1 = m_2 = k_1 = k_2 = 0$ .

It is possible to show directly, as in Section 2, that the map  $(s, t) \mapsto (s, t) * p$  is an embedding of  $\mathbb{C}^2$  into  $(\mathbb{C}^*)^4$ . This also follows from the next proposition. Let  $\pi : (\mathbb{C}^*)^4 \rightarrow T$  be the holomorphic map defined by

$$\pi(x, y, z, w) = \left[ ze^{-(\lambda^1 \log x + \mu^1 \log y)}, we^{-(\lambda^2 \log x + \mu^2 \log y)} \right].$$

PROPOSITION 3.1. — *The map  $\pi$  is the quotient map for the action  $*$ . That is to say,*

$$\pi(x, y, z, w) = \pi(x', y', z', w') \iff (s, t) * (x, y, z, w) = (x', y', z', w')$$

for some  $(s, t) \in \mathbb{C}^2$ .

*Proof.* — Suppose  $\pi(x, y, z, w) = \pi(x', y', z', w')$ . Then

$$\frac{z'}{z} = e^{[\lambda^1(\log(x'/x) + 2\pi\sqrt{-1}c) + \mu^1(\log(y'/y) + 2\pi\sqrt{-1}d)]}$$

and

$$\frac{w'}{w} = e^{[\lambda^2(\log(x'/x) + 2\pi\sqrt{-1}c) + \mu^2(\log(y'/y) + 2\pi\sqrt{-1}d)]}$$

for some integers  $c$  and  $d$ . The reader can observe that it is possible to choose a single branch of the logarithm so that all the numbers appearing in these equations make sense. Now let  $s = \log(x'/x) + 2\pi\sqrt{-1}c$  and  $t = \log(y'/y) + 2\pi\sqrt{-1}d$ . Then

$$\frac{x'}{x} = e^s \quad \text{and} \quad \frac{y'}{y} = e^t,$$

and so  $(s, t) * (x, y, z, w) = (x', y', z', w')$ . □

From general theory of Lie group actions, it follows that the bundle  $\pi : (\mathbb{C}^*)^4 \rightarrow T$  is a locally trivial  $\mathbb{C}^2$  bundle. However, we will show this more directly.

To this end, let  $D = D_1$  be a fundamental domain of  $T$  in  $\mathbb{C}^2$ , e.g.,  $D$  is the convex hull of the 16 vertices  $\{b_1e_1 + b_2e_2 + b_3\lambda + b_4\mu \mid b_1, b_2, b_3, b_4 \in \{0, 1\}\}$ , and let  $D_2, \dots, D_N$  be translates of  $D$  in  $\mathbb{C}^2$  such that

$$\overline{D} \subset \bigcup_{j=1}^N D_j.$$

Let  $\mathcal{D}_j$  be the image of  $D_j$  in  $\mathbb{C}^* \times \mathbb{C}^*$  under the map  $(z, w) \mapsto (e^{2\pi\sqrt{-1}z}, e^{2\pi\sqrt{-1}w})$ . The restriction to  $\mathcal{D}_j$  of the projection  $p : \mathbb{C}^* \times \mathbb{C}^* \rightarrow T$

is biholomorphic onto its image  $\Delta_j$ . The bundle structure of  $(\mathbb{C}^*)^4 \rightarrow T$  is now defined as follows. Let  $Y_j = \pi^{-1}(\Delta_j) \subset (\mathbb{C}^*)^4$  and let  $\varphi_j : \Delta_j \times \mathbb{C}^2 \rightarrow Y_j$  be given as follows. Suppose  $(\zeta, \eta) \in D_j$ . Then

$$(3) \quad \varphi_j([\zeta, \eta], (s, t)) = (e^s, e^t, e^{\lambda^1 s + \mu^1 t} \zeta, e^{\lambda^2 s + \mu^2 t} \eta).$$

This map is well defined because  $D_j$  is a fundamental domain, and thus  $D_j$  contains a unique  $(\zeta, \eta)$  projecting onto  $[\zeta, \eta]$ .

It can be verified that the map  $\varphi_j$  is biholomorphic, but we will actually write down the inverse. To this end, fix  $\xi = (x, y, z, w) \in \pi^{-1}(\Delta_j)$ , and choose a branch of log such that  $\log x$  and  $\log y$  are well defined. Define the integers  $m = m_j(\xi)$  and  $n = n_j(\xi)$  to be those integers such that

$$(e^{-(\lambda^1 \log x + \mu^1 \log y + 2\pi\sqrt{-1}(\lambda^1 m + \mu^1 n))} z, e^{-(\lambda^2 \log x + \mu^2 \log y + 2\pi\sqrt{-1}(\lambda^2 m + \mu^2 n))} w) \in D_j.$$

Then

$$\begin{aligned} \varphi_j^{-1}(\xi) = & \left[ e^{-(\lambda^1 \log x + \mu^1 \log y + 2\pi\sqrt{-1}(\lambda^1 m + \mu^1 n))} z, \right. \\ & \left. e^{-(\lambda^2 \log x + \mu^2 \log y + 2\pi\sqrt{-1}(\lambda^2 m + \mu^2 n))} w \right], \\ & \log x + 2\pi\sqrt{-1}m_j(\xi), \log y + 2\pi\sqrt{-1}n_j(\xi). \end{aligned}$$

We leave it to the reader to verify that  $\varphi_j^{-1}$  is well defined. The main thing is that  $\varphi_j^{-1}$  is continuous, even though the chosen branch of log, as well as  $m$  and  $n$ , are not.

It follows from this discussion that the transition functions  $g_{ij} = \varphi_j^{-1} \circ \varphi_i$  for  $\pi$  are of the form

$$(4) \quad g_{ij}([z, w])(s, t) = (s + 2\pi\sqrt{-1}m_{ij}, t + 2\pi\sqrt{-1}n_{ij})$$

for some integers  $m_{ij}$  and  $n_{ij}$ . In particular, they are locally constant. We summarize this as follows.

**PROPOSITION 3.2.** — *The fiber bundle  $\pi : (\mathbb{C}^*)^4 \rightarrow T$  is affine and flat.*

The transition functions  $g_{ij}$  for the affine bundle above can be used to construct a vector bundle  $E \rightarrow T$  whose transition functions are given by

$$G_{ij}([z, w]) \begin{pmatrix} r \\ s \\ t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2\pi\sqrt{-1}m_{ij} & 1 & 0 \\ 2\pi\sqrt{-1}n_{ij} & 0 & 1 \end{pmatrix} \begin{pmatrix} r \\ s \\ t \end{pmatrix}.$$

Evidently the projectivization  $X := \mathbb{P}(E)$  of  $E$  is a  $\mathbb{P}_2$  bundle over  $T$ . Moreover, even though the coordinate functions  $r, s, t$  are not globally defined, the divisor  $D = (r = 0) \subset X$  is well defined, i.e.,

$$G_{ij}([z, w]) \begin{pmatrix} 0 \\ s \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ s \\ t \end{pmatrix}.$$

From this, we also see that  $D$  is a trivial  $\mathbb{P}_1$ -bundle over  $T$ . Since the projectivization of a vector bundle over a compact Kähler manifold is itself Kähler,  $X$  is Kähler. The proof of Theorem 1 is complete.

#### 4. Proof of Theorem 2.

Let us return to the affine bundle  $\pi : (\mathbb{C}^*)^4 \rightarrow T$  in Proposition 3.2. Though somewhat abusive, we will also denote the  $\mathbb{P}_2$ -bundle  $X \rightarrow T$  by  $\pi$ .

LEMMA 4.1. — *The affine bundle  $\pi$  does not have a section or an affine subbundle of rank 1.*

*Proof.* — A section of  $\pi$  gives a compact complex torus in  $(\mathbb{C}^*)^4$ , which contradicts the maximum principle. Under the local trivialization  $\varphi_j : \Delta_j \times \mathbb{C}^2 \rightarrow Y_j \subset (\mathbb{C}^*)^4$  in (3), an affine subbundle is given by a linear equation

$$a_j + b_j s + c_j t = 0$$

where  $a_j, b_j, c_j$  are holomorphic functions on  $\Delta_j$ . The transition functions (4) give the relations

$$a_i + b_i s + c_i t = a_j + b_j(s + 2\pi\sqrt{-1}m_{ij}) + c_j(t + 2\pi\sqrt{-1}n_{ij}).$$

We then have that  $b_j$  and  $c_j$  define global holomorphic functions on  $T$ . Thus they are constant and the functions  $a_j$  on  $\Delta_j$  satisfy

$$a_i - a_j = bm_{ij} + cn_{ij}.$$

Thus the  $\mathbb{Z}$ -valued cocycles  $\{m_{ij}\}$  and  $\{n_{ij}\}$  become linearly dependent in  $H^1(T, \mathcal{O})$ .

But the  $\mathbb{C}^2$ -bundle  $\pi$  is precisely the quotient of the trivial  $\mathbb{C}^2$ -bundle on the universal cover  $\mathbb{C}^2$  of  $T$  where  $\gamma \in \Gamma$  acts by  $(p, q) \mapsto (p + \gamma, q + \gamma)$ . Thus the two cocycles are linearly independent.  $\square$

For the rest of this section, we assume that  $T$  has no nonconstant meromorphic functions or curves, and that every line bundle on  $T$  is flat. This is true for a general choice of  $T$ .

LEMMA 4.2. — *There cannot be two algebraically independent meromorphic functions on  $X$ .*

*Proof.* — To obtain a contradiction, suppose that  $f$  and  $g$  are two independent meromorphic functions on  $X$ . Since  $T$  has no nonconstant meromorphic function, possibly after perturbing  $f$  and  $g$ , we can assume that there is an irreducible component  $Z$  of the variety  $(f = g = 0)$  whose intersection with the generic fiber of  $\pi$  is a finite set disjoint from  $D$ , the compactifying divisor. Let  $\mathcal{A} \subset T$  be the set of points  $t$  such that either  $\pi^{-1}(t) \cap Z$  is not finite, or else  $\pi^{-1}(t) \cap Z \cap D \neq \emptyset$ . Since  $\mathcal{A}$  is a proper analytic subvariety of  $T$ , it must be finite.

For  $t \in T - \mathcal{A}$ , let  $\zeta_t$  be the center of mass of the set-with-multiplicity  $\pi^{-1}(t) \cap Z$ , and let  $Z'$  be the set  $\{\zeta_t \mid t \in T - \mathcal{A}\}$ . Then  $Z'$  is a holomorphic section of the  $\mathbb{C}^2$ -bundle  $\mathbb{P}(E) - D = (\mathbb{C}^*)^4$  over  $T - \mathcal{A}$ , and thus extends to a section of  $\mathbb{P}(E) - D$  over  $T$  by Hartogs extension, a contradiction to Lemma 4.1.  $\square$

Let us say that a meromorphic function  $f$  on  $M$  has fiberwise linear levels if for each  $c \in \mathbb{P}_1$ , the level sets  $(f = c)$  intersect the fibers of  $\pi$  in hyperplanes.

LEMMA 4.3. — *If  $f$  is a nonconstant meromorphic function on  $X$ , then  $f$  has fiberwise linear levels.*

*Proof.* — Note first that the  $\mathbb{C}^2$ -action (2) on  $(\mathbb{C}^*)^4$  extends holomorphically to an action on  $X$ , which fixes  $D$  pointwise. Moreover, the action preserves fibers and is linear on them. If  $f$  is a meromorphic function on  $X$  with non-linear fiber levels, then by pulling back  $f$  with the  $\mathbb{C}^2$  action, we could produce a second meromorphic function  $g$  with different level foliation on the fibers. Thus  $g$  and  $f$  would be algebraically independent, contradicting Lemma 4.2.  $\square$

An easy consequence of the assumption that every line bundle on  $T$  is flat is

LEMMA 4.4. — *Let  $L$  be a line bundle on  $T$ . If there exists a non-zero map of line bundles  $L \rightarrow \mathcal{O}$ , then  $L = \mathcal{O}$ .*

Now we can complete the proof of Theorem 2 by

LEMMA 4.5. — *The manifold  $X$  has no meromorphic functions with fiberwise linear levels.*

*Proof.* — A level set of such a meromorphic function defines a rank-2 subbundle  $F \subset E$  such that  $\mathbb{P}F \neq D$ . From the transition functions, we have the exact sequence

$$0 \longrightarrow \mathcal{O}^2 \longrightarrow E \longrightarrow \mathcal{O} \longrightarrow 0.$$

By Lemma 4.4,  $F$  must surject to  $\mathcal{O}$  and  $\mathbb{P}F \cap (\mathbb{P}E - D)$  defines a rank-1 affine subbundle of  $\pi : (\mathbb{C}^*)^4 \rightarrow T$ , a contradiction to Lemma 4.1.  $\square$

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