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An $L^p$-version of a theorem of D.A. Raikov


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1. Introduction.

Let $G$ be a locally compact group, for $p \in (1, \infty)$, let $Pf_p(G)$ denote the closure of $L^1(G)$ in the convolution operator norm on $L^p(G)$. Denote by $W_p(G)$ the dual of $Pf_p(G)$ which is contained in the space of pointwise multipliers of the Figà-Talamanca Herz space $A_p(G)$. (See [5], [8], [9] for all this.)

It is shown in these notes that on the unit sphere of $W_p(G)$ the weak * (i.e. the $\sigma(W_p, Pf_p)$ topology and the $A_p$-multiplier topology coincide ($u_\beta \rightarrow u$ in the latter if $\|(u_\beta - u)v\| \rightarrow 0$ for each $v \in A_p(G)$).

If $p = 2$ and $G$ is amenable then $W_2(G)$ is just the Fourier-Stieltjes algebra of $G$, denoted $B(G)$, and $A_2(G)$ is the Fourier algebra of $G$. From this point of view the above enunciation is an $L^p$-version of a theorem of D.A. Raikov, which asserts that on the positive face of the unit sphere of $B(G)$ the weak * topology coincides with the topology of uniform convergence on compact sets (since $A_p(G)$ always contains functions which take the value one on a given compact set the latter topology is clearly weaker than the $A_p(G)$ multiplier topology; and on norm bounded sets obviously stronger than the weak * topology).

The proof is based on a technique of G.C. Rota [10], first used in harmonic analysis by E.M. Stein; our application is close to the work of M. Cowling [3]. On the other hand this paper continues the line of studies taken up by E.E. Granirer and M. Leinert in [7].

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2. An estimate for the $L^p$-operator norm of the sum of two “spectrally disjoint” operators.

If $R$ and $S$ are two commuting normal (of course bounded) operators on an Hilbert space $H$ then, via the Gelfand transform, $R$ and $S$ correspond to some continuous functions on a locally compact space $X$; further $R, S$ are spectrally disjoint, if the supports of those functions are disjoint. It then follows easily that $\| R + S \| = \max \{ \|R\|, \|S\| \}$; we remark that there exists an orthogonal projection $P$ with $PR = R = RP$ and $(1 - P)S = S = S(1 - P)$.

From this:
$$\|(R + S)\xi\| = \|(R + S)(P + 1 - P)\xi\|$$
$$= \|PRP\xi + (1 - P)S(1 - P)\xi\|$$
$$= (\|PRP\xi\|^2 + \|(1 - P)S(1 - P)\xi\|^2)^{1/2}$$
$$\leq (\|R\|^2 \|P\xi\|^2 + \|S\|^2 \|(1 - P)\xi\|^2)^{1/2}$$
$$\leq \max \{ \|R\|, \|S\| \} (\|P\xi\|^2 + \|(1 - P)\xi\|^2)^{1/2}$$
$$\leq \max \{ \|R\|, \|S\| \} \|\xi\| \text{ for all } \xi \in H.$$

Now let $(X, \mu)$ be a $\sigma$-finite measure space; an operator $T$ acting on all $L^p$-spaces will be called special if:

i) $Tf \geq 0$ if $f \geq 0$

ii) $\|Tf\|_p \leq \|f\|_p \quad f \in L^p(X, \mu), 1 \leq p \leq \infty$

iii) $T1_X = 1_X$

iv) $\int_X Tf(x) \overline{g(x)} \, d\mu(x) = \int_X f(x) \overline{Tg(x)} \, d\mu(x) \quad f, g \in L^2(X, \mu)$.

Those operators will serve as a substitute for orthogonal projections, since by a method due to G.C. Rota they may be seen as conditional expectations on a certain measure space.
We begin with the following observation:

**Proposition.** Let \((Y, \mathcal{F}, \nu)\) be a \(\sigma\)-finite measure space, \(\mathcal{F}_1 \subset \mathcal{F}\) a sub-\(\sigma\)-algebra of \(\mathcal{F}\) such that \((Y, \mathcal{F}_1, \nu)\) is again \(\sigma\)-finite, (which ensures the existence of a conditional expectation operator \(E_1\) with respect to \(\mathcal{F}_1\)).

Then we have for \(\xi, \eta \in L^p(Y, \mathcal{F}, \nu)\):

\[
\|E_1 \xi + (1 - E_1) \eta\|_p \leq (\|\xi\|_p^p + 2 \|\eta\|_p^p)^{1/r}
\]

where \(r = p\) if \(1 < p < 2\) and \(r = p'\), the index conjugate to \(p\), if \(2 < p < \infty\).

**Proof.** Clearly

1) \(\|E_1 \xi + (1 - E_1) \eta\|_1 \leq \|\xi\|_1 + 2 \|\eta\|_1\)

2) \(\|E_1 \xi + (1 - E_1) \eta\|_2^2 \leq \|\xi\|_2^2 + \|\eta\|_2^2 \leq \|\xi\|_2^2 + 2 \|\eta\|_2^2\)

3) \(\|E_1 \xi + (1 - E_1) \eta\|_\infty \leq \|\xi\|_\infty + 2 \|\eta\|_\infty\)

and the assertion follows from interpolation between 1) and 2) (resp. 2) and 3)) on mixed \(L^p\)-spaces (see [1]).

Let \((X, \mu)\) and \(T\) be as above. Define \(Y = X \times X\) and endow \(Y\) with the usual product \(\sigma\)-algebra denoted \(\mathcal{F}\). We define a measure \(\nu\) on \(Y\) by requiring that

\[
\nu(S_0 \times S_1) = \int_X \chi_{S_0}(x) T \chi_{S_1}(x) \, d\mu(x)
\]

(whenever \(S_0, S_1\) are measurable subsets of \(X\)).

Denote by \(\mathcal{F}_1\) and \(\mathcal{F}_0\) the \(\sigma\)-algebras of sets \(X \times S (S \subseteq X\) measurable), respectively of sets \(S \times X\) (\(S \subseteq X\) measurable), further denote by \(E_1, E_0\) the corresponding conditional expectation operators. For a measurable function \(\xi\) on \(X\) we define for \(x = (x_0, x_1) \in Y\)

\[
\xi^i(x_0, x_1) = \xi(x_i) \quad i = 0, 1.
\]

Then \(\xi \rightarrow \xi^0\) gives rise to an isometric isomorphism between \(L^p(X, \mu)\) and the subspace of \(\mathcal{F}_0\)-measurable elements of \(L^p(Y, \nu)\); whereas \(\xi \rightarrow \xi^1\), from \(L^p(X, \mu)\) to \(L^p(Y, \nu)\), does not increase norms.
Further:
\[
E_0(\xi^i) = \begin{cases} 
(T\xi)^0 & \text{if } i = 1 \\
\xi^0 & \text{if } i = 0 
\end{cases}
\]

\[
E_1(\xi^0) = (T\xi)^1.
\]

For a proof of these facts we refer the reader to the book of E.M. Stein [11].

**Proposition.** — Let \((X, \mu)\) be a \(\sigma\)-finite measure space, \(T\) a special operator and \(1 < p < \infty\). Then for \(\xi_1, \xi_2 \in L^p(X, \mu)\):

\[
\|T^2\xi_1 + (1 - T^2)\xi_2\| \leq (\|\xi_1\|_p^p + 2\|\xi_2\|_p^p)^{1/r}, \text{ with } r = \min(p, p').
\]

**Proof.** — We apply the above procedure to \(T\), then

\[
\|T^2\xi_1 + (1 - T^2)\xi_2\| = \|E_0((T\xi_1)^1 + \xi_2^0 - (T\xi_2)^1)\|
\leq (\|T\xi_1\|_r^r + \xi_2^0 - (T\xi_2)^1\|)
= \|E_1(\xi^0_1) + (1 - E_1)(\xi^0_2)\|
\leq (\|\xi_1^0\|^r + 2\|\xi_2^0\|^r)^{1/r}.
\]

**Corollary.** — Let \(R, S\) be bounded operators on \(L^p(X, \mu)\), then we have

\[
\|T^2R + (1 - T^2)S\| \leq (\|R\|^r + 2\|S\|^r)^{1/r}.
\]

3. The weak* topology on the unit sphere of \(W_p(G)\).

Let \(G\) be a locally compact group, with a fixed left Haar measure \(dg\) and modular function \(\Delta\). Let \(L^p(G), 1 \leq p \leq \infty\), denote the usual Lebesgue spaces with respect to \(dg\) and for functions \(f, h\) on \(G\) let be defined \(f * h(x) = \int_G f(g)h(g^{-1}x) \, dg\),

\[
f \sim(g) = f(g^{-1})\Delta(g^{-1}), f^* = \bar{f} \sim, f^\nu(g) = f(g^{-1}).
\]

For this section let now \(p \in (1, \infty)\) be fixed and let \(A_p(G)\) (as in [8]) be the algebra of functions \(u\) on \(G\) which can be represented as \(u = \sum_{n=1}^\infty v_n * w_n^\nu\), where

\[
\sum_n \|v_n\|_p \cdot \|w_n\|_{p\leftarrow \infty}, \frac{1}{p} + \frac{1}{p'} = 1.
\]
The norm on $A_p$ is defined as the inf $\Sigma \|v_n\|_p, \|w_n\|_p$ taken over all such representations of $u$.

If $f$ is an element of $L^1(G)$ then on one hand $w \mapsto f * w$ defines a convolution operator on $L^p(G)$ and on the other $u \mapsto \int_G f(g) u(g) \, dg$ a continuous linear functional on $A_p(G)$. From $\langle f, v * w^\vee \rangle = \langle f * w, v \rangle$ it follows that the corresponding norms of $f$ coincide.

Let $\mathcal{P}_p(G)$ denote the closure of $L^1(G)$ in the algebra of convolution operators on $L^p(G)$ and $W_p(G)$ the dual space of $\mathcal{P}_p(G)$, which is contained in $L^\infty(G)$, and in which $A_p(G)$ is norm non-increasingly embedded.

If $t$ is a nonnegative (almost everywhere) function with $\|t\|_1 = 1$ then $t * t^\sim$, as a convolution operator, is almost a special operator, except that $(G, dg)$ might not be $\sigma$-finite.

Let $U_\alpha$ be an open relatively compact neighborhood base at the identity $e$ of $G$. If $V_\alpha = V_\alpha^{-1}$ are open neighborhoods of $e$ such that $V_\alpha^2 \subset U_\alpha$ then $\tau_\alpha = \lambda(V_\alpha)^{-1} \chi_{V_\alpha}$, where $\lambda(V)$ denotes the Haar measure of $V$ and $\chi_V$ its characteristic function, $t_\alpha = \tau_\alpha * \tau_\alpha^\sim$ and $e_\alpha = t_\alpha * t_\alpha$ are approximate identities for $L^1(G)$, $e_\alpha$ being the square of a "special" operator. This last fact we seem really to need in the proof of the following

**Lemma.** — Let $e_\alpha = t_\alpha * t_\alpha$ be as above, if $u_\beta$ is a net in $W_p(G)$ such that $u_\beta \rightarrow u_0$ in the weak* topology of $W_p(G)$ and if $\|u_\beta\|_{W_p} \rightarrow \|u_0\|_{W_p}$, then for $\varepsilon > 0$ there exist $\beta_0, \alpha_0$ such that

i) $\|e_{\alpha_0} * u_\beta - u_\beta\|_{W_p} \leq \varepsilon$ for all $\beta \geq \beta_0$

and

ii) $\|e_{\alpha_0} * u_0 - u_0\|_{W_p} \leq \varepsilon$.

**Proof.** — Clearly ii) is a consequence of i), so it is enough to prove i) and we may assume that $\|u_0\| = 1$. We suppose now that there is a net $u_\beta$ which converges to $u_0$ as described in the lemma and an $\varepsilon > 0$ such that for all $\alpha_0, \beta_0$ there exists $\beta > \beta_0$ with

$\|e_{\alpha_0} * u_\beta - u_\beta\| > \varepsilon$.

We shall derive a contradiction.
Let $0 < \eta < \varepsilon/2$, to be specified later, and choose $f \in L^1(G)$ with
\[ \| f \|_{P_F^p} = 1, \langle f, u_0 \rangle \geq 1 - \eta. \]
then choose $\alpha_0$ with
\[ \| e_{\alpha_0} \ast f - f \|_{P_F^p} \leq \eta \]
and $\beta_0$ with
\[ |\langle u_\beta, e_{\alpha_0} \ast f \rangle - \langle u_0, e_{\alpha_0} \ast f \rangle| \leq \eta, \]
\[ \| u_\beta \| \leq 1 + \eta \text{ for all } \beta \geq \beta_0. \]
We may now fix $\beta > \beta_0$ with
\[ \| e_{\alpha_0} \ast u_\beta - u_\beta \|_{\omega} > \epsilon \]
and find $g \in L^1(G), \| g \|_{P_F^p} = 1$, with
\[ \langle e_{\alpha_0} \ast u_\beta - u_\beta, g \rangle > \epsilon - \eta \]
i.e. $\langle u_\beta, (e_{\alpha_0} - 1) \ast g \rangle = \langle u_\beta, (1 - e_{\alpha_0}) \ast (-g) \rangle > \epsilon - \eta$.
Now, the supports of $t_{\alpha_0}, f, g$ are contained in a $\sigma$-finite open subgroup $G_0$ of $G$. Since for an $L^1(G)$ function $h$ with support in $G_0: \| h \|_{P_F^p(G_0)} = \| h \|_{P_F^p(G)}$, we may apply the estimation of the corollary of the last section to $e_{\alpha_0} \ast f - \lambda g + \lambda e_{\alpha_0} \ast g$, where $\lambda > 0$:
\[ \| e_{\alpha_0} \ast f + (1 - e_{\alpha_0}) \ast (-\lambda g) \| \leq (\| f \| + 2 - \lambda g \|)^{1/r} = (1 + 2\lambda r)^{1/r}. \]
So on one hand
\[ \langle u_\beta, e_{\alpha_0} \ast f + (1 - e_{\alpha_0}) \ast (-\lambda g) \rangle \leq \| u_\beta \| (1 + 2\lambda r)^{1/r} \]
\[ \leq (1 + \eta)(1 + 2\lambda r)^{1/r}, \]
and on the other
\[ |\langle u_\beta, e_{\alpha_0} \ast f + (1 - e_{\alpha_0}) \ast (-\lambda g) \rangle| = |\langle u_0, f \rangle + \langle u_0, e_{\alpha_0} \ast f - f \rangle \]
\[ + \langle u_\beta - u_0, e_{\alpha_0} \ast f \rangle + \lambda \langle e_{\alpha_0} \ast u_\beta - u_\beta, g \rangle| \geq 1 - 3\eta + \lambda \varepsilon/2. \]
But $1 - 3\eta + \lambda \varepsilon/2 \leq (1 + \eta)(1 + 2\lambda r)^{1/r}$ cannot hold for all $\eta \in (\varepsilon/2, 0), \lambda > 0$.

We thank the referee for pointing out to us the following implication of the lemma (due to M. Cowling, theorem 3 of [3]; see [4] for a different proof).
COROLLARY. — Translations act continuously on $W_p(G)$.

Proof. — For $h \in G$ let $u(g) = u(h^{-1}g)$ and $u_h(g) = u(gh)$, $g \in G$.

We first consider left translations, if $u$ is in $W_p(G)$, $\varepsilon > 0$ then we find, by the lemma, an element $e$ of $L^1(G)$ with

$$\|e * u - u\|_{W_p} \leq \varepsilon.$$ 

Then

$$\|h u - u\|_{W_p} \leq \|h u - h(e * u)\|_{W_p} + \|h(e * u) - e * u\|_{W_p}$$

$$+ \|e * u - u\|_{W_p}$$

$$\leq \|u - e * u\|_{W_p} + \|h e - e\|_1 \|u\|_{W_p} + \|e * u - u\|_{W_p}$$

$$\leq 3 \varepsilon$$ if $h$ is in a neighborhood $V$ of the identity, choosen such that $\|h\|_{L^1(G)} < \varepsilon$ for all $h \in V$.

From $\|f\|_{p_{f_p}} = \|f\|_{p_{f'_p}}$, for $f \in L^1(G)$, we infer that $\|u\|_{W_p} = \|u\|_{W_{p'}}$ for $u \in W_p(G)$, and hence the continuity of right translations, on $W_p$, follows from that of left translations on $W_{p'}$.

It has been proved by Herz [8], that for $v \in A_p(G)$ and $u \in W_p(G)$ the pointwise product $u \cdot v$ is in $A_p(G)$ and $\|u \cdot v\|_{A_p} \leq \|u\|_{A_p} \|v\|_{A_p}$.

We say that a net $u_\beta \in W_p(G)$ converges to $u \in W_p$ in the $A_p$-multiplier topology, if, for all $v \in A_p$, $u_\beta v \to uv$ in $A_p$ norm.

THEOREM. — On the unit sphere $S = \{u \in W_p/\|u\|_{W_p} = 1\}$ of $W_p(G)$ the weak* and the $A_p$-multiplier topology coincide.

Proof. — Let $u_\beta, u \in S$ be such that $u_\beta \to u$ in the weak* topology. Let $e_\alpha = t_\alpha * t_\alpha$ be as in the lemma. Then for $v \in A_p(G)$

$$\|u_\beta v - uv\| \leq \|(u_\beta - e_{\alpha_0} * u_\beta) v\| + \|e_{\alpha_0} * (u_\beta - u)\| v\|$$

$$+ \|(e_{\alpha_0} * u - u) v\|$$

$$\leq \varepsilon \|v\| + \|e_{\alpha_0} * (u_\beta - u)\| v\| + \varepsilon \|v\|,$$

when $\beta \geq \beta_0$, where $\alpha_0, \beta_0$ are choosen according to the lemma.

Since $t_{\alpha_0} \in L^1(G) \cap L^\infty(G)$ has compact support we may
apply lemma 6 of [7] and find $\beta_1 \geq \beta_0$ such that for $\beta \geq \beta_1$
\[ \| [e_{\alpha} \ast (u_\beta - u)] v \| \leq \varepsilon. \]

For the converse it is sufficient to note that $u_\beta \rightarrow u$
uniformly on compact sets, whenever $u_\beta \rightarrow u$ in the $A_p$-multiplier
topology and $\|u\|_{w_p}$ is bounded. So, for a compact set $K$, let $v \in A_p^p(G)$ be a function which takes the value one on $K$
(e.g. take $v = \lambda(U)^{-1} \chi_u \ast \chi_{K^{-1}}$, where $U$ is open, relatively compact) then
\[
\sup_{g \in K} |(u_\beta - u)(g)| \leq \| (u_\beta - u) v \|_\infty \leq \| (u_\beta - u) v \|_{A_p} \rightarrow 0.
\]

The following corollary is of interest with respect to the
problems considered in [6]. To state it, let, for a compact set
$K \subset G$, $A_K^p(G) = \{ v \in A_p^p(G)/\text{supp } v \subset K \}$. This space we consider as a subspace of $W_p(G)$.

**COROLLARY.** - On the unit sphere of $(A_K^p(G), \| \cdot \|_{w_p})$ the
weak * and the norm topology coincide.

**Proof.** - Let $u_\beta, u \in A_K^p(G)$ be such that $u_\beta \rightarrow u$
in the weak * topology and $\|u_\beta\|_{w_p} = 1 = \|u\|_{w_p}$. Then, for $v \in A_K^p(G)$ which is constant one on $K$,
\[
\|u_\beta - u\|_{w_p} = \| (u_\beta - u) v \|_{w_p} \leq \| (u_\beta - u) v \|_{A_p} \rightarrow 0
\]
by our theorem. The converse is evident.

4. Addendum.

When the paper was already finished we realized that, by our
method, we can improve a theorem of E.E. Granirer, theorem 3
of [6], which we think to be central in the cited paper.

Let $M_A^p(G)$ be the algebra of (continuous, bounded)
functions on $G$ which pointwise multiply $A_p(G)$ into itself and
let for $u \in M_A^p(G)$ $\|u\|_{M_A^p} = \sup \{ \|uv\|_{A_p}/\|v\|_{A_p} = 1 \}$.

**THEOREM.** - Let $u \in M_A^p(G)$ be such that $u(g) = \|u\|_{M_A^p}$
for an $g \in G$. If $u_\beta$ is a net in $M_A^p(G)$ such that
\[
\|u_\beta\|_{M_A^p} \rightarrow \|u\|_{M_A^p}
\]
and $u_\rho \to u$ in the $\sigma(\mathcal{MA}_p(G), L^1(G))$-topology then $u_\rho \to u$ in the $A_p$-multiplier topology.

To prove this theorem we need an auxiliary result for whose proof we use that we admit complex scalars for our linear spaces.

**Proposition.** The linear span of \{v \in A_p(G) | v(e) = \|v\|_{A_p}, v has compact support\} is norm dense in $A_p(G)$.

**Proof.** The dual space of $A_p(G)$ is the ultra weak operator topology closure of $Pf_p(G)$ in the space of bounded operators on $L^p(G)$, the duality is given by

$$\langle T, u \rangle = \sum_{n=1}^{\infty} \int_G Tw_n(g) v_n(g) \, dg$$

when $u = \sum_{n=1}^{\infty} v_n \ast w_n^* \in A_p(G), T \in A_p(G)^*$ (see [9]).

By theorem 4.1 and theorem 9.4 of [2] we have

$$e^{-1} \|T\| \leq \sup \{ \langle Tf, f^* \rangle | f \in L^p(G), \|f\|_p = 1 \},$$

where $f^* = |f|^{p-1} \exp(-i \arg(f(.)))$ is the unique element of $L^{p'}(G)$ with $\langle f, f^* \rangle = 1$ and norm one.

If we approximate $f \in L^p(G)$ by $f \cdot \chi_K$, where $K \subseteq G$ is a suitable compact set, in the $L^p$-norm, then $(f \chi_K)^* = f^* \chi_K$ approximates $f^*$ in $L^{p'}$-norm. This is why we can restrict the supremum to be taken over the elements $f \in L^p(G)$ with compact support and norm one.

If $f \in L^p(G)$ has compact support then $v = f^* \ast f^v$ will have compact support too, and if $\|f\|_p = 1$ then,

$$1 = \|f\|_p \|f^*\|_{p'} \geq \|v\|_{A_p} \geq \|v\|_{\infty} = f^* \ast f^v(e) = \|f\|_p^p = 1.$$ 

Hence for any $T \in A_p(G)^*$ :

$$e^{-1} \|T\| \leq \sup \{ \langle T, v \rangle | v(e) = \|v\|_{A_p}, v \text{ has compact support} \},$$

and the proposition follows by an application of the Hahn-Banach theorem.
Proof of the theorem. — We may assume \( \|u\|_{\text{MA}_p} = 1 \) and, since translations are isometries of \( \text{MA}_p(G) \), we may further assume \( u(e) = \|u\|_{\text{MA}_p} = 1 \).

Since there exists \( \beta_0 \) such that \( \sup \{ \|u_{\beta}\|_{\text{MA}_p} / \beta \geq \beta_0 \} < \infty \) it suffices, by the above proposition, to show \( u_\beta v \to uv \) when \( v \) has compact support, say \( K \), and \( v(e) = \|v\|_{\text{A_p}} = 1 \). Now, the \( u_\beta v \) and \( uv \) are elements of \( \text{A}_p(K) \), and on this space the \( \text{W}_p \)-norm is equivalent to the \( \text{A}_p \)-norm (this follows from proposition 1 of [6] and proposition 3 of [8]). Thus we must only show \( \|u_\beta v - uv\|_{\text{W}_p} \to 0 \).

Clearly, \( u_\beta v \to uv \) in the weak\(^*\) topology of \( \text{A}_p(K) \), and, if we can show that \( \lim \|u_\beta v\|_{\text{W}_p} = \|uv\|_{\text{W}_p} \), then the corollary of the last section finishes the proof.

But,
\[
1 = u(e)v(e) \leq \|uv\|_{\text{W}_p} \leq \lim \inf \|u_\beta v\|_{\text{W}_p}
\]
and
\[
1 = u(e)v(e) = \|u\|_{\text{MA}_p} \|v\|_{\text{A}_p} = \lim \|u_\beta\|_{\text{MA}_p} \|v\|_{\text{A}_p}
\]
\[
\geq \lim \sup \|u_\beta v\|_{\text{A}_p} \geq \lim \sup \|u_\beta v\|_{\text{W}_p}
\]
from which \( \lim \|u_\beta v\|_{\text{W}_p} = 1 = \|uv\|_{\text{W}_p} \) follows.

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