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THE INFORMATION CONTENT OF INTEREST-RATE OPTION PRICES

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Abstract

In this paper the information content of option prices is analysed in three different ways: i) by testing the hypothesis of absence of predictability; ii) comparing the implied volatility derived from option prices with actual volatility and forecasts from GARCH models. As the first test is concerned option price changes are found to be serially correlated because so is the volatility of the underlying asset and, due to this reason, serial correlation does not enhance arbitrage profits. In the other case, the various measures of volatility differ markedly in the short run and none emerges as the best performer, though, over the medium run, such discrepancies are no longer statistically significant.

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1. Introduction

If the assumptions underpinning BLACK and SCHOLES (1973, hereafter BS) model held true, the principle that arbitrage profits are absent in efficient markets would be sufficient to evaluate equilibrium option prices. However, there is overwhelming evidence that the assumption of constant variance is inappropriate, coming both from the prices of underlying assets – which unambiguously display time-varying second moments – and from the options prices themselves – which give rise to the well-known *volatility smile* and *wave* patterns when the volatility measures they imply are plotted against moneyness and time-to-maturity (BOSSAERT and HILLION, 1995).

Abandoning the hypothesis that the variance is constant poses momentous challenges. To finance theorists, who have been struggling in the arduous search for option pricing formulae accommodating the presence of time-varying second moments; efforts in this direction – which will be briefly recalled in the next Section – have mainly relied either on the hypothesis that the variance follows a specific law of motion and volatility risk is not priced (SCOTT, 1987; JOHNSON and SHANNO, 1987; HULL and WHITE or, in discrete-time frameworks, on the assumption that the return variances are time-dependent according to a generalised autoregressive conditional heteroskedasticity (GARCH) scheme (DUAN, 1995).

To market makers, who must quote option prices without knowing either the true generating process of return variances or the correct formula to be applied once the stochastic law of motion of second moments is known; casual experience shows that hiring top econometricians and mathematicians to crunch random price kernels is not sufficient: attention is also paid to the (noisy) information embodied in price and non-price variables as well as to strategic considerations on the behaviour of competitors in the financial arena.

To economists, who have to explore the implications on asset prices of the behaviour of (possibly risk-averse) option issuers under conditions of time-varying volatility; since option prices come to be determined by equilibrium rather than arbitrage considerations, the powerful tool of risk-neutral valuation seems no longer appropriate, thereby challenging the validity of standard results in asset pricing theory.[2]

These examples are sufficient to give a flavour of the broad scope of the empirical and theoretical research agenda stimulated by the relaxation of some of the simplifying assumptions of standard finance theory. Against this background, the paper chooses a particular focus to tackle the issues involved with option pricing when second moments are not constant: the assessment of the information content of option prices in order to retrieve

2. The possibility that there is no suitable risk-neutral measure representation for prices can also arise from agents' aversion to pervasive "Knightian" uncertainty (EPSTEIN and WANG, 1995).

market participants' perceptions of current asset-price volatility as well as their expectations about its future developments. This approach can shed some light on the usefulness of option prices as indicators in the conduct of monetary policy. Its empirical findings, however, have a more general bearing on the relationship between expectations formation and asset price determination. Indeed, *expectations* of volatility are an inescapable ingredient of economic agents' portfolio choices aiming at the desired balance in the trade off between risk and return.

Extraction of information about expectations on volatility involves particular difficulties. First, contrary to the case of the *level* of asset returns, the actual, realised value of volatility is difficult to compare with expectations – even for the single operator who knows (or should know) his own thoughts. A drastic change in the first moment of the price of an asset, in fact, may well be the result of a shock of extraordinary large dimensions (an improbable draw) when the variance of the fundamentals (the whole distribution) is unchanged and thus is consistent with the previously-held expectations about volatility. Secondly, sample variance is not necessarily the best benchmark to evaluate the properties of volatility expectations: agents' perceived risk is related to conditional higher-order moments, i.e. forecast variance after the cost-effective gathering and use of available information helpful to predict asset returns.

Options are the assets whose price is more directly linked to expected volatility and thus they stand as the prime source of information for the issue at hand. Our sample consists of daily observations, over the period from September 1990 to October 1995, about options traded at LIFFE on futures for US, German, Italian ten-year Treasury bonds and for three-month eurodeposits denominated in US dollars and Deutsche marks; data are obtained from LIFFE. For each of these financial assets, three different kinds of option have been considered:

a) the closest to be at-the-money, whose price enjoys desirable analytical properties when the variance of the underlying asset price is stochastic;
b) the most liquid, which should be the least influenced by any market imperfection;
c) the ones with the shortest residual life, so as to reduce the number of changes in contracts implied in the definition of homogeneous price series.

The empirical analysis of the paper pursues two main objectives. First, to assess the efficiency of option prices in the wake of the long standing tradition in finance (e.g. FAMA, 1970): only if markets are efficient, i.e. prices embody the relevant information quickly and consistently, can they be reliable indicators. Secondly, to explore the statistical and economic properties of the measure of volatility embodied in option prices (hereafter implied volatility) through a comparison with sample variance and recursive out-of-sample forecasts from GARCH models which are consistent with implicit variance as regards both the available information set and the time horizon to which the variance refers (see LAMOUREUX and LASTRAPES, 1993).

The paper is organised as follows. Next Section summarises the theoretical issues involved in option pricing when the second moment of the underlying asset price is time-varying. Section 3 evaluates the efficiency of option prices by testing the predictive content of lagged option prices and exchange and interest rates as well as of other non-price variables, such as traded volumes, suggested by the recent literature on the microstructure of financial markets. Section 4 investigates the properties of different measures of actual and expected volatility. Section 5 concludes.

2. Stochastic variance and the price of options

In the framework pioneered by MERTON (1973) and BLACK and SCHOLES (1973), option prices are a function of observable variables independent of agents' risk preferences. If trade occurs continuously in perfectly competitive markets, options are redundant assets and their value can be derived by arbitrage arguments with reference to a portfolio composed of the risk-free and the underlying assets. If the interest rate is constant, assets do not pay dividends over the remaining life of the option (τ) and the price of the underlying asset (S) follows a geometric Brownian motion, defined as:

$$\mathrm{d}S_t = \alpha S_t \mathrm{d}t + \sigma S_t \mathrm{d}z_t, \tag{1}$$

the price of a call option with exercise price K is equal to:

$$C(S, \sigma^2, \tau | K, r) = SN(\mathrm{d}_1) - Ke^{-r\tau} N(\mathrm{d}_2) \tag{2}$$

where "|" denotes conditioning, σ^2 is the variance of the logarithmic rates of change of the underlying asset, N(.) is the Normal cumulative density function, d_1 and d_2 are given by:

$$\mathrm{d}_1 = \frac{\left(\log\left(\frac{S}{R}\right) + \left(r + 0.5\sigma^2\right)\tau\right)}{\left(\sigma\tau^{0.5}\right)} \tag{3}$$

$$\mathrm{d}_2 = \mathrm{d}_1 - \sigma\tau^{0.5} \tag{4}$$

Even initial empirical applications of this formula (BLACK and SCHOLES, 1972) showed a significant divergence between market and theoretical prices, especially for the options which were in- and out-of-the-money. Moreover, estimates for the variance obtained by inverting the above formula (i.e. identifying the value of σ^2 which minimises the difference between the theoretical and the market option prices) displayed remarkable variability over time.

Subsequently, a vast body of research, stimulated by the path-breaking papers of ENGLE (1982) and BOLLERSLEV (1986), highlighted the instability of the second moments of most macroeconomic variables as well as the unforeseeable succession of periods of calm and turbulence, particularly for financial variables (see BOLLERSLEV *et al.*, 1994, for a survey).

In principle, BLACK and SCHOLES analysis can be easily extended to the case of stochastic volatility as the arbitrage argument is robust to the relaxation of the hypothesis of constant second moment for the price of underlying asset. In practice, no analytical solution has been found, requiring the resort to approximations or simulations (see HULL and WHITE, 1987; HESTON, 1993a; SCOTT, 1987; JOHNSON and SHANNO, 1987). This is apparent in the model put forward by SCOTT (1987) which rests on the assumption that the price of the underlying asset and its standard deviation follow a bivariate stochastic process:

(5) $$\mathrm{d}S_t = \alpha S_t \mathrm{d}t + S_t \sigma_t \mathrm{d}Z_{1,t}$$

(6) $$\mathrm{d}\sigma_t = \beta(-\sigma_t + \bar{\sigma})\mathrm{d}t + \lambda Z_{2,t}$$

Since there are two sources of risk in the model, the application of the arbitrage argument requires two risky assets, in addition to the risk-free rate in order to build a portfolio with the suitable characteristics of risk and return. If two call options with different time-to-maturity are used as risky assets, the risk premium related to movements of the conditional variance is neglected and correlation between the two Brownian motions is assumed to be zero, the value of the underlying at maturitity can be shown to be equal to

(7) $$S_T = S_0 \exp\left[\int_0^t \left(r - 0.5\,\sigma^2(s)\right) \mathrm{d}s + \int_0^t \sigma(s)\,\mathrm{d}\,Z_{1,s}\right],$$

so that the conditional distribution $(S_t | S_0, \sigma_0)$ is log-normal, with the following characteristics:

(8) $$E\left[S_1 | S_0, \sigma_s\right] = S_0\, e^{-rt}$$

(9) $$\ln \frac{S_t}{S_0} \approx N(rt - 0.5v, v) \text{ with } v = \int_0^t \sigma^2\,(s)\mathrm{d}\,s.$$

This, together with the results in COX *et al.* (1985), implies that the price of the call option can be written as

(10) $$C(S_t, \sigma_t^2, \tau) = \int_0^\infty \left[S_0 N(\mathrm{d}_1) - K e^{-r\tau} N(\mathrm{d}_2)\right] \mathrm{d}\,F(v),$$

an expression which admits no analytical solution and has to be calculated by numerical integration.

Instead, the approach put forward by HULL and WHITE (1987) rests on the hypothesis that the price of the underlying and its variance[3] – rather than the standard deviation – follow the bivariate process:

3. This hypothesis is particularly relevant for the simulation of option prices by GARCH schemes since it ensures that, in continuous time, the variance of the variance is linear in σ_k, a condition which, as shown by NELSON and FOSTER (1994), is necessary to obtain consistent estimates of the true underlying volatility.

$$\mathrm{d}S_t = \phi S_t \mathrm{d}t + S_t \sigma_t \mathrm{d}Z_{1,t} \tag{11}$$

$$\mathrm{d}\sigma_t^2 = \mu\sigma_t^2 \mathrm{d}t + \zeta\sigma_t^2 \mathrm{d}Z_{2,t} \tag{12}$$

Instead of the usual arbitrage argument, resort is made to the results by GARMAN (1976) on the differential equation satisfied by the price of an asset whose return depends upon two state variables. Under the hypothesis that the risk deriving from the variability of the second moment is not priced and that the two sources of uncertainty are not correlated, the price of a call option can be shown to be

$$C(S_t, \sigma_t^2, \tau) = \int BS(\sigma_t^{2*})\, h(\sigma_t^{2*}|I_t)\mathrm{d}\sigma_t^{2*} \tag{13}$$

where BS denotes the BLACK and SCHOLES formula, $h(.)$ is the density of the distribution of the conditional variance and σ_t^{2*} the average variance in the remaining life of the option, is equal to:

$$\sigma_t^{2*} = \frac{1}{\tau}\int_t^T \sigma^2(s)\mathrm{d}s. \tag{14}$$

This expression, which, again, has no analytical solution, shows that, when the variance is stochastic, the option price coincides with the expected value of the price under constant variance, discounted at the average variance over the life to maturity of the option.

CHIRAS and MANASTER (1978) were the first to notice that the ex-post sample variance is a better predictor of option prices than the sample variance measured at the time options are written and that implied variance helps predict option prices more accurately than lagged sample variance. This evidence has provided the background to the recent stream of research resorting to the application of GARCH schemes to option pricing. Even though derived in discrete time, GARCH processes converge to stochastic differential equation when the sampling frequency shrinks to zero (NELSON, 1990). Thus, they approximate financial models developed in continuous time, enabling their evaluation from an empirical standpoint; for example, the model by HULL and WHITE can be discretised by a GARCH(1,1) scheme.

Supposing that the utility function of a representative agent belongs to the constant relative risk aversion (CRRA) class and that the single stochastic process in the economy obeys a GARCH(p,q) process, DUAN (1995) shows that the equilibrium asset price and its variance evolve according to the following stochastic process defined in terms of risk-neutral probability [4]

4. So is called a measure of probability that, while leaving unchanged the ordering of the probabilities of elementary events, ensures that the evolution of discounted equilibrium prices follows a martingale.

$$\log \frac{S_t}{S_{t-1}} = r - 0.5\,\sigma_t^2 + \xi_t \tag{15}$$

$$\xi_t \,|\, \Phi_{t-1} \approx N(0, \sigma_t^2) \tag{16}$$

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j\, \xi_{t-j}^2 + \sum_{i=1}^{p} \beta_i\, \sigma_{t-i}^2 \tag{17}$$

where Φ_t denotes the set of information available at t.

These assumptions imply that, at expiration, the value of the underlying asset is

$$S_T = S_0 \exp\left[\tau\, r - 0.5 \sum_{s=t+1}^{T} \sigma_s^2 + \sum_{s=t+1}^{T} \xi_s\right] \tag{18}$$

and that the price of the option is:

$$C_t = e^{-r\tau}\, E^Q \left[\max\big[(S_T - K), 0\big] \,|\, \Phi_t\right] \tag{19}$$

where Q denotes a transformation of the original probability.

In this case too, a closed-form solution is not available. This approach is consistent with the high excess kurtosis for the distributions of underlying asset returns, which characterises the available evidence for the vast majority of financial variables (NELSON, 1990). Another analytical advantage of the approach is that it allows the explicit assessment of different hypotheses on the distribution of the returns on the underlying asset. In particular, recent contributions (NELSON, 1990; FORNARI and MELE, 1997) suggest that the logarithmic rates of change of the prices of financial assets can be reasonably approximated by a General Error distribution, a density function which belongs to the Gamma family and encompasses the Normal and the Laplace as special cases. The computation of option prices under this distribution can be performed within the GARCH approach.

3. Efficiency

The classic definition of informational efficiency (FAMA, 1970) – i.e. the rapid reflection of available information in asset prices – can be also applied to option prices as the first step in assessing their reliability as indicators of (perceived) variability. FAMA defines three forms of efficiency, corresponding to three different information sets available to economic agents: **weak efficiency**, which rules out risk-free profits from an investment strategy derived from the lagged prices of the asset in question; **semi-strong efficiency**, which extends the condition of the previous definition to all the information in public domain; **strong efficiency**, which refers to an even broader set of information, including private one.

Before moving to econometric testing of option market efficiency, it may be worth recalling that informational efficiency is a less restrictive condition than market "perfection". In addition to informational efficiency, the notion of market perfection requires atomistic competition, which makes all economic agents *price-takers*, as well as the absence of transaction costs. Informational efficiency is compatible with the presence of imperfections of other nature, even though prices fully reflect available information, consistently with market expectations. However, if option issuers enjoy a significant degree of market power, other factors besides expectations on volatility come to have a bearing on option prices, including issuers' risk preferences as well as the distribution of wealth in the economy. When the variance of the logarithmic changes of the price of the underlying asset is time dependent, not only there is no closed-form solution for option pricing, but, as a result of that, option issuing becomes an inherently risky activity which can only be hedged imperfectly. As a result, if the option-issuing industry is not fully competitive and volatility is stochastic, risk-neutral valuation is unlikely to be an adequate approach, with wide-ranging implications for asset pricing which have not yet been explored.

In practice, the evaluation of market efficiency generally boils down to testing that the time series of price changes cannot be forecast using the available information, thus ruling out risk-free profits. In other words, econometric analysis takes the form of parametric testing whether the first differences of option prices are orthogonal to a set of information which includes lagged values of the first difference of option prices themselves (weak efficiency) as well as of the prices of some financial assets, such as exchange and interest rates (semi-strong efficiency).

The predictive power of lagged variables should be ideally tested with reference to time series of option prices which are observed continuously, so to avoid that "jumps" due to changes in the option contract defining the statistical observation generate results which risk being erroneously interpreted as indicators of market inefficiency. Unfortunately, the average life of option contracts is too short to generate sufficient observations for econometric analysis to be based on the prices of a single contract. In order to reduce the number of changes in the reference option within the sample, the efficiency analysis considers not only observations referring to the most liquid and the most at-the-money options but also data on options which are closest to maturity.

In addition to the lagged values of option prices as well as of exchange and short-term interest rates, market efficiency is assessed with reference to other non-price variables: traded volume of options, the number of outstanding contracts, the differential between the maximum and the minimum price. Such variables are likely to convey additional information about the existence of market imperfections. Indeed, their role in the determination of option prices is suggested by the recent literature on market microstructure (see, e.g., LYONS, 1994; O'HARA, 1994) which put forward new behavioural hypotheses on asset trading in non-perfectly competitive markets. Imperfections may be short-lived, albeit recurrent – as for the temporary monopoly power, originating from heterogeneous information flows in a high-frequency setting – or may be structural as they derive from barriers to entry in the financial industry.

Against this background, the specification of the regression equation to test efficiency is quite general so as to increase the power of the tests, that is:[5]

5. As outlined by a referee, it is very hard to perform a semi-strong test of efficiency; it is instead very simple to run a weak-form test of this kind. Ex-post, i.e. after examining the reults of the regressions, our intention was indeed to perform a simple weak-form efficiency tests, which revealed the unusual negative autocorrelation feature of options price changes. However, we had rather keep the other regressors, with five lags being determined according to the Akaike information criterion. These evidenced the absence of arbitrage opportunities, however well expected if one thinks about the importance of LIFFE: it would be extremely hard to make money without risk on this market. The regressors, as outlined by the referee, were 66 in each regression: however, sample size was 1293 for Bund and Euromark options and 1003 for Btp options, with samples ranging from 3/9/90 to 11/10/95 in the first two cases and from 11/10/91 to 11/10/95 in the latter case. Observations were daily and data were provided by LIFFE.
As concerns the generating process of the data, all of the variables in the regression were integrated of the first order. That is why they apper in first difference in the regression. We did not consider a cointegration term in the efficiency regressions. This happens since the "independent" variables are clearly predetermined with respect to options prices. Also if this were not the case, it would be extremely hard to test cointegration on more than 10 variables; in fact, the cointegrating vectors would be hard to identify, without economic reasons for any restrictions. Error correction terms would then be still easy to impose, but they would lack, in our mind, significance.

$$
\begin{aligned}
(20) \quad \Delta C_t^s = {} & \mu + \sum_{j=1}^{5} a_j \Delta C_{t-j}^{L} + \sum_{j=1}^{5} b_j \Delta C_{t-j}^{ATM} + \sum_{j=1}^{5} c_j \Delta C_{t-j}^{CL} \\
& + \sum_{j=1}^{5} d_j \Delta DMUSA_{t-j} + \sum_{j=1}^{5} e_j \Delta YENUSA_{t-j} + \sum_{j=1}^{5} f_j \Delta FFDM_{t-j} \\
& + \sum_{j=1}^{5} g_j \Delta r_{t-j} + \sum_{j=1}^{5} h_j CVAR_{t-j} + \sum_{j=1}^{5} m_j \Delta Vol_{t-j}^{ATM} + \sum_{j=1}^{5} n_j \Delta Vol_{t-j}^{L} \\
& + \sum_{j=1}^{5} p_j \Delta Vol_{t-j}^{Cl} + \sum_{j=1}^{5} q_j OI_{t-j} + \sum_{j=1}^{5} t_j REV_{t-j} + \varepsilon_t^s
\end{aligned}
$$

where the superscript s denotes the three types of options taken into consideration – i.e. L (most liquid), CL (closest-to-maturity), ATM (closest to be at-the-money) – and

$DMUSA$	Deutsche Mark/Dollar exchange rate;
$YENUSA$	Yen/Dollar exchange rate;
$FFDM$	French Franc/Deutsche Mark exchange rate;
r	short-term interest rate for the relevant currency;
$CVAR$	range of price quotes during business hours;
Vol^{ATM}	traded volume for the option closest to be at the money;
Vol^{L}	traded volume for the most liquid option;
Vol^{CL}	traded volume for the option closest to maturity;
OI	number of outstanding options;
REV	price revision occurring when the market is closed;
ε_t^s	zero-mean, uncorrelated error term.

The above equation has been estimated for option contracts regarding futures on the three-month eurodeposit in Deutsche Marks as well as on Bund and BTP, for which enough information is available. For each of these underlying assets, the analysis has been replicated for the price of the three kinds of option (s= L, ATM, CL).

Tables 1a-c show the results of the estimation, reporting only the significant coefficients for the sake of brevity.

Table 1a

Efficiency test – Bund

Estimated equation

$$
\begin{aligned}
\Delta C_t^s = \mu &+ \sum_{j=1}^{5} a_j \Delta C_{t-j}^{L} + \sum_{j=1}^{5} b_j \Delta C_{t-j}^{ATM} + \sum_{j=1}^{5} C_{t-j}^{CL} \\
&+ \sum_{j=1}^{5} d_j \Delta DMUSA_{t-j} + \sum_{j=1}^{5} e_j \Delta DYENUSA_{t-j} + \sum_{j=1}^{5} f_j \Delta DFFDM_{t-j} \\
&+ \sum_{j=1}^{5} g_j \Delta r_{t-j} + \sum_{j=1}^{5} h_j CVAR_{t-j} + \sum_{j=1}^{5} m_j \Delta Vol_{t-j}^{ATM} + \sum_{j=1}^{5} n_j \Delta Vol_{t-j}^{L} \\
&+ \sum_{j=1}^{5} p_j \Delta Vol_{t-j}^{CL} + \sum_{j=1}^{5} q_j OI_{t-j} + \sum_{j=1}^{5} t_j REV_{t-j} + \varepsilon_t^s
\end{aligned}
$$

Most liquid option (C^L)- R^2=0.36 DW=2.03

significant coefficients

a_1=-0.746 (-26.20)	b_1=0.283 (2.52)	c_1=-0.026 (-2.25)	d_1=2.181 (2.10)			g_1=0.361 (1.97)				p_1=-0.002 (-2.35)		t_1=0.408 (2.47)
a_2=-0.651 (-18.65)	b_2=0.373 (2.76)	c_2=-0.024 (-2.00)										
a_3=-0.447 (-11.88)	b_3=0.358 (2.53)											
a_4=-0.253 (-7.25)												
a_5=-0.134 (-4.71)												

At-the-money option (C^{ATM})- R^2=0.37 DW=2.02

significant coefficients

a_1=0.014 (1.82)	b_1=-0.709 (-23.75)							m_1=4.8E-6 (1.93)				
a_2=0.027 (2.89)	b_2=-0.509 (-14.16)						h_2=0.136 (4.39)					
	b_3=-0.307 (-8.18)							m_3=6.9E-6 (2.73)				
	b_4=-0.222 (-6.30)						h_4=-0.139 (-4.56)					
	b_5=-0.112 (-3.94)											

Closest-to-maturity option (C^{CL})- R^2=0.15 DW=2.01

significant coefficients

		c_1=-0.320 (-10.59)		e_1=-0.090 (-2.20)						p_1=0.015 (6.24)		t_1=1.885 (4.29)
		c_2=-0.079 (-2.49)		e_2=0.079 (1.92)			h_2=0.692 (2.23)			p_2=0.008 (3.13)		t_2=0.852 (1.93)
							h_3=-0.725 (-2.32)					t_3=-1.020 (-2.29)
							h_5=0.632 (2.04)					

Table 1b

Efficiency test – D-Mark Eurodeposits

Estimated equation

$$
\begin{aligned}
\Delta C_t^s = \mu &+ \sum_{j=1}^{5} a_j \Delta C_{t-j}^{L} + \sum_{j=1}^{5} b_j \Delta C_{t-j}^{ATM} + \sum_{j=1}^{5} C_{t-j}^{CL} \\
&+ \sum_{j=1}^{5} d_j \Delta DMUSA_{t-j} + \sum_{j=1}^{5} e_j \Delta DYENUSA_{t-j} + \sum_{j=1}^{5} f_j \Delta DFFDM_{t-j} \\
&+ \sum_{j=1}^{5} g_j \Delta r_{t-j} + \sum_{j=1}^{5} h_j CVAR_{t-j} + \sum_{j=1}^{5} m_j \Delta Vol_{t-j}^{ATM} + \sum_{j=1}^{5} n_j \Delta Vol_{t-j}^{L} \\
&+ \sum_{j=1}^{5} p_j \Delta Vol_{t-j}^{CL} + \sum_{j=1}^{5} q_j OI_{t-j} + \sum_{j=1}^{5} t_j REV_{t-j} + \varepsilon_t^s
\end{aligned}
$$

Most liquid option (C^L)- R^2=0.41 DW=2.05

significant coefficients

a_1=-0.782 (-27.79)		c_1=0.190 (2.36)				g_1=0.338 (3.90)						
a_2=-0.430 (-12.29)												
a_3=-0.315 (-8.76)												
a_4=-0.299 (-8.58)	b_4=-0.623 (-2.40)			e_4=0.014 (2.07)								
a_5=-0.215 (-7.68)						g_5=0.176 (2.14)						

At-the-money option (C^{ATM})- R^2=0.38 DW=2.02

significant coefficients

	b_1=-0.766 (-26.77)											
a_2=0.030 (2.60)	b_2=-0.526 (-14.85)											t_2=-1.9E-2 (-3.33)
	b_3=-0.361 (-9.71)											
	b_4=-0.262 (-7.38)							m_4=2.5E-6 (2.57)				
	b_5=-0.122 (-4.24)							m_5=-2.1E-6 (-2.17)				

Closest-to-maturity option (C^{CL})- R^2=0.01 DW=2.00

significant coefficients

	b_1=-0.170 (-2.21)				f_1=0.475 (1.92)							
a_2=-5.9E-2 (-1.86)												
a_3=-5.8E-2 (-1.83)										p_3=9.8E-4 (3.21)		
						g_5=6.7E-2 (2.21)						

Table 1c

Efficiency test – BTP

Estimated equation

$$\Delta C_t^s = \mu + \sum_{j=1}^{5} a_j \Delta C_{t-j}^{L} + \sum_{j=1}^{5} b_j \Delta C_{t-j}^{ATM} + \sum_{j=1}^{5} C_{t-j}^{CL}$$
$$+\sum_{j=1}^{5} d_j \Delta DMUSA_{t-j} + \sum_{j=1}^{5} e_j \Delta DYENUSA_{t-j} + \sum_{j=1}^{5} f_j \Delta DFFDM_{t-j}$$
$$+\sum_{j=1}^{5} g_j \Delta r_{t-j} + \sum_{j=1}^{5} h_j CVAR_{t-j} + \sum_{j=1}^{5} m_j \Delta\, Vol_{t-j}^{ATM} + \sum_{j=1}^{5} n_j \Delta\, Vol_{t-j}^{L}$$
$$+\sum_{j=1}^{5} p_j \Delta\, Vol_{t-j}^{CL} + \sum_{j=1}^{5} q_j OI_{t-j} + \sum_{j=1}^{5} t_j REV_{t-j} + \varepsilon_t^s$$

Most liquid option (C^L)- R^2=0.45 DW=2.02

significant coefficients

a_1=-0.911 (-27 40) a_2=-0.768 (-17 19) a_3=-0.512 (-9.98) a_4=-0.320 (-5 99) a_5=-0.133 (-2.98)	b_2=-0.397 (-2.06)	c_5=0.086 (2.25)					h_5=0 406 (2.20)					t_1=0 145 (2.36)

At-the-money option (C^{ATM})- R^2=0.28 DW=2.00

significant coefficients

a_1=0 017 (2 36)	b_1=-0.606 (-17 21) b_2=-0.461 (-11.16) b_3=-0.321 (-7.64) b_4=-0.144 (-3.63)	c_1=-0.025 (-3.05) c_3=-0.021 (-2.58)		e_5=-1.507 (-2.05)	f_5=-1.988 (-2.10)		h_3=-0 094 (-2.35)	m_2=7.8E-5 (2.33)		p_1=7.7E-3 (2.70)	q_5=7.3E-6 (2.14)	t_5=-0.024 (-2 54)

Closest-to-maturity option (C^{CL})- R^2=0.08 DW=2.00

significant coefficients

				e_1=-0.117 (-2.61)	f_2=8.278 (2.09)	g_1=0.238 (1.92)	h_1=0.314 (1 90)			p_3=0 031 (2.52) p_4=0.029 (2.33)		t_1=0 117 (2.14)

A large, statistically significant proportion of the movements in option prices can be explained by lagged variables, particularly the dependent variable: the coefficients of determination of the regressions range from 15 percent for the Bund, in the case of the closest-to-maturity (CL) option, to 45 percent for the BTP, in the case of the most liquid option. Only two equations – namely those for eurodeposits in Deutsche Marks and for BTP, closest-to-maturity option – show low coefficients of determination (1 and 8 percent, respectively).

While for other financial markets the finding that price changes are autocorrelated would immediately imply a verdict of inefficiency, this conclusion does not seem warranted for options in a setting characterised by stochastic volatility.

Option prices are an increasing function of the variance of the price of the underlying asset, consistently with the intuition that hedging the risk of variation of a price which fluctuates in a wide interval is more costly than for a less variable price. If the variance is not constant but follows a stationary, autocorrelated process, the price of an option on that asset will also be autocorrelated, given that, other things being equal, (autoregressive) changes in the variance are sufficient to bring about movements in equilibrium option prices. The pattern of autocorrelation in option prices, however, need not exactly mirror that in the variance of the price of the underlying asset since the relationship between the two variables is non-linear.

As reasserted by the estimates presented in Section 4, the volatility of the prices of the assets underlying the options considered in this analysis presents a strong and highly significant degree of autocorrelation which stands as the most likely factor accounting for the serial correlation in the first (log) differences of option prices. More specifically, the first five lags of the dependent variable are highly significant with negative coefficients, consistently with agents' expectations of a gradual mean-reversion of volatility after a shock,[6] shown by the econometric analysis of next Section.

This evidence, however, cannot be interpreted as a proof of market inefficiency since the possibility to forecast future option price changes does not always entail risk-free profits. In particular, if the autocorrelation of option prices only reflects the autocorrelation of the variance of the price of the underlying asset, purchases (or sells) of options aimed at exploiting the predictability of option prices will yield no sure profits because of the associated risk stemming from the unpredictable variations in the price of the underlying assets. Indeed, taking a position in options is equivalent to taking a position (at time t) in the underlying asset. While the benefit from such a strategy is related to the forecast of option prices at time $t+1$ (which in turn is related to the volatility which will prevail then), the risk due to unpredictable changes in the price of the underlying is in relation with volatility at time t. Therefore, either the

6. Mean-reversion is also consistent with the stationarity of volatility, in line with the typical pattern of empirical findings for macroeconomic variables (BOLLERSLEV, 1994).

strategy aimed at exploiting the predicted option price is carried out without hedging the risk of movements in the price of the underlying – and hence will not yield risk-free profits – or, conversely, this risk is hedged by an appropriate portfolio whose cost depends on the variance of the underlying asset at time t, not to its forecast for the next period, thereby eliminating the profitability of a strategy which exploits the predictive content of option prices.

This argument suggests that the forecastability of option prices deriving from the serial correlation of the variance does not reveal the existence of market frictions which prevent economic agents from carrying out riskless arbitrage activities. Rather, it is the implication (to our knowledge so far unnoticed) of well-known statistical properties of second moments of asset prices which opens no margin to (unexploited) arbitrage opportunities. This conclusion is strengthened when one performs a test for the joint significance of the coefficients of the independent variables: the hypothesis that they are equal to zero cannot be rejected.[7]

4. The information content of the implied variance

This section investigates the information content of implied variance – the measure of dispersion derived inverting the traditional Black-and-Scholes formula – through a comparison with the estimates of a GARCH model and with sample variance, defined as the mean of the squared logarithmic rates of change of the price of the underlying asset in the remaining life of the option.

Following DAY and LEWIS (1992) and LAMOUREUX and LASTRAPES (1993), the first step in the analysis is testing the hypothesis that implied variance provides no information in addition to that obtainable from the past values of the price of the underlying asset. More specifically, the test is carried out within the framework of a GARCH(1,1) scheme, where the stationary and uncorrelated residual (ε_t) of an autoregressive representation for the first difference of the price of the underlying asset (denoted by H_t in the model in Table 2) is assumed to have a variance, σ_t^2, which evolves through time according to the following process:

$$(21) \qquad \sigma_t^2 = b_1 + b_2\, \varepsilon_{t-1}^2 + b_3\, \sigma_{t-1}^2$$

with $b_1 > 0$ and $b_2,\ b_3 \geq 0$ so that the variance is definite positive.

If the implied variance, denoted by $\left[\sigma^{imp}\right]$ were a sufficient statistic for the second moment of the price changes of the underlying asset, its coefficient, when added as an additional regressor in (21), should equal one and the coefficients of the original GARCH scheme should no longer be statistically significant, i.e.:

$$(22) \qquad b_2 = 0;\ b_3 = 0;\ b_4 = 1 \qquad \text{in} \qquad \sigma_t^2 = b_1 + b_2\varepsilon_{t-1}^2 + b_3\sigma_{t-1}^2 + b_4[\sigma_{t-1}^{imp}]|\,.$$

7. In the nine cases under consideration (three types of option for each of the three assets) the test under the null that the coefficients of the independent variables – others than the lag of the dependent variable – are equal to zero ranges between 1.11 and 2.47, so that the hypothesis cannot be rejected.

Table 2 **Information content of the implied variance**

All the coefficients are statistically significant at the 5 percent level of confidence, with the exception of those denoted by an asterisk (*t*-ratios) are omitted for brevity).

Estimated model

$$H_t = a_1 + a_2 H_{t-1} + \varepsilon_t$$
$$\varepsilon_t | I_{t-1} \approx N(0, \sigma_t^2)$$
$$\sigma_t^2 = b_1 + b_2 \varepsilon_{t-1}^2 + b_3 \sigma_{t-1}^2 + b_4 \left[\sigma_{t-1}^{imp}\right]^2 + b_5 V_{t-1}$$

where H denotes the logarithmic change of the price of the underlying asset, σ^{imp} the implied standard deviation, v the traded volume, I_t the information set.

	a_1	a_2	$\sqrt{b_1}$	$\sqrt{b_2}$	$\sqrt{b_3}$	b_4	b_5	log of likelihood
BTP								
• standard model ($b_4 = b_5 = 0$)	6.96E-4	-0.045	3.37E-4	0.595	0.639			3542.82
• implied variance ($b_5 = 0$)	1.25E-4*	-0.010*	6.13E-4	0.327	0.908	0.092		4578.77
• traded volume ($b_4 = 0$)	3.67E-5*	-0.034*	2.69E-4	0.273	0.949		4.16E-5	4529.25
• full model	1.20E-4*	-0.137*	4.16E-4	0.322	0.905	0.096	1.44E-5*	4581.64
BUND								
• standard model ($b_4 = b_5 = 0$)	9.52E-4	0.155	3.13E-4	0.734	0.594			5210.08
• implied variance ($b_5 = 0$)	2.31E-4	-0.193	3.29E-4	0.245	0.958	0.025		6494.56
• traded volume ($b_4 = 0$)	2.46E-4	-0.194	3.10E-4	0.273	0.959		2.53E-6	6410.15
• full model	1.87E-4*	-0.189	1.64E-6*	0.274	0.924	0.112	1.97E-6*	6507.09
T-BOND								
• standard model ($b_4 = b_5 = 0$)	1.77E-3	0.107	3.40E-4	0.505	0.727			2007.26
• implied variance ($b_5 = 0$)	3.83E-4	-0.015*	1.99E-4*	0.141*	0.188*	0.765		3232.40
• traded volume ($b_4 = 0$)	3.27E-4	-0.021*	5.70E-3	0.278	0.016*		-4,86E-5	3218.56
• full model	3.78E-4*	-0.013*	1.78E-3*	0.208	4.82E-3	0.733	-3.53E-5	3235.22
D-Mark eurodeposits								
• standard model ($b_4 = b_5 = 0$)	-1.86E-4	-0.225	1.39E-3	0.274	0.892			3522.02
• implied variance ($b_5 = 0$)	4.58E-5	-0.052	1.51E-3	0.273	0.702	0.374		5297.94
• traded volume ($b_4 = 0$)	1.40E-5	-0.034	1.81E-3	0.302	0.897		1.72E-4	5127.83
• full model	-1.24E-4	-0.053	-2.27E-4	0.308	0.545	0.725	4.81E-4	5387.18
Dollar eurodeposits								
• standard model ($b_4 = b_5 = 0$)	3.61E-4	-0.075	0.0072	0.448	0.683			2119.57
• implied variance ($b_5 = 0$)	4.99E-4	-0.158	6.46E-3	0.457	0.692	0.187		2595.05
• traded volume ($b_4 = 0$)	1.42E-4*	-0.090	7.00E-3	0.412	0.708		0.049	2147.51
• full model	3.67E-4	-0.171	6.28E-3	0.427	0.714	0.176	4.91E-3	2605.10

These restrictions are tested within a general framework which encompasses the hypothesis that the traded volume of options influences the variance of the underlying asset for reasons first put forward by TAUCHEN and PITTS (1983) and recently discussed, among others, by ANDERSEN (1996).

Table 2 (page 16) shows the results of the estimation for the five assets considered in the analysis, starting from the restricted model in which the traded volume and the implied volatility are not included, extending the model with the addition of each variable separately and then including both regressors.

The parameters of the GARCH equation are significant in all cases, with a persistence, measured by $b_2 + b_3$, which is always below unity and ranges from 0.67 for the Dollar eurodeposits to 0.89 for the Bund. Conditional variances are thus stationary processes, which revert to their long-run values after a shock, even though at different speeds.

Implied variance is a significant regressor for all assets and specifications and thus has informative power, as testified by the higher value of the likelihood function obtained when the variable is included in the model. However, the restrictions implied by the null hypothesis that implied variance is a sufficient statistic for sample variance (that is $b_2 = b_3 = 0$ and $b_4 = 1$) are strongly rejected in all cases (the values of the test are omitted from Table 2 for sake of brevity). These results unambiguously show that implied variance does not incorporate all the information about the variance which can instead be obtained from the past record of prices changes in the underlying asset.

With the exception of the full models for the Bund and BTP, traded volume is always a regressor with a coefficient significantly different from zero. This variable has thus a specific informative content which, however, does not overlap with that of lagged sample variance and implied variance: the coefficients of such regressors remain significantly different from zero even when traded volume is added to the specification.

When comparing the behaviour over time of the various measures of variance, plotted in Figures 1a-b, it is important to recall that implied variance and GARCH estimates reflect two different concepts of riskiness: the former concerns movements of the price of the underlying asset during the whole life-to-maturity of the option; the latter measure, instead, refers only to the current, single point in time. Moreover, fitted values from GARCH models depend upon an information set spanning the whole sample, and thus embody information unknown when setting option prices from which the implied variance is derived. Therefore an unbiased comparison requires the correction both of the difference in the time horizon spanned by the option and of the heterogeneity of the information set underpinning the two measures. Following LAMOUREUX and LASTRAPES (1993), implied variance is therefore compared with the recursive, τ-step-ahead forecasts based on a GARCH model estimated using only the observations available at each point in time, with τ, chosen on the basis of the remaining life of the option. The comparability between implied variance and GARCH estimates is thus ensured both in terms of information set (recursive aspect of the estimation) and time horizon (τ-step-ahead aspect).[8]

The statistical evaluation of the differences between the various measures is based on three indices which quantify the divergence between implied and GARCH variances as well as the divergence of these two measures with sample variance, as measured by the mean of the squared logarithmic rates of change of the price of the underlying asset in the remaining life of the option, i.e.

$$\hat{\sigma}_t = \left[\frac{1}{N}\sum_{k=1}^{N} \varepsilon_{t+k}^2\right] \tag{23}$$

The indices are defined below while their mean values and standard errors are reported in **Table 3**.[9]

$$\varepsilon_1 = \sigma_t^{imp} - \sigma_{t+k} \,|\, t \tag{24}$$

$$\varepsilon_2 = \sigma_t^{imp} - \hat{\sigma}_t \tag{25}$$

$$\varepsilon_3 = \sigma_{t+k} \,|\, t - \hat{\sigma}_t \tag{26}$$

Figures 1a-b evidence sizeable differences between implied and GARCH measures of volatility, which tend to become less systematic in the recent period, as suggested by their greater dispersion at the end of the sample. An analogous behaviour can be observed for the difference between implied and GARCH measures and sample variance, which are large but not systematic.

8. DAY and LEWIS (1992) compare the implicit variance with its one-step-ahead values, obtained from a recursive estimation of the GARCH model; the two measures refer to different time horizons even if they share the same information set.

9. Other indicators, such as the mean error or the absolute mean error, provided the same information. As concerns the models reported in Table 2, the referee suggests to discriminate between the models by means of a likelihood ratio test. On this point, the likelihood gains between competing models are so large that the test can be carried out visually. However, our intention was not that of discriminating between models; rather we wanted to show that the inclusion of the implied variance or the traded volume of the option in the GARCH equation did not bring the GARCH coefficient to nil, which implies that the volatility concepts do not fully overlap.

Figure 1a **Measures of variability**

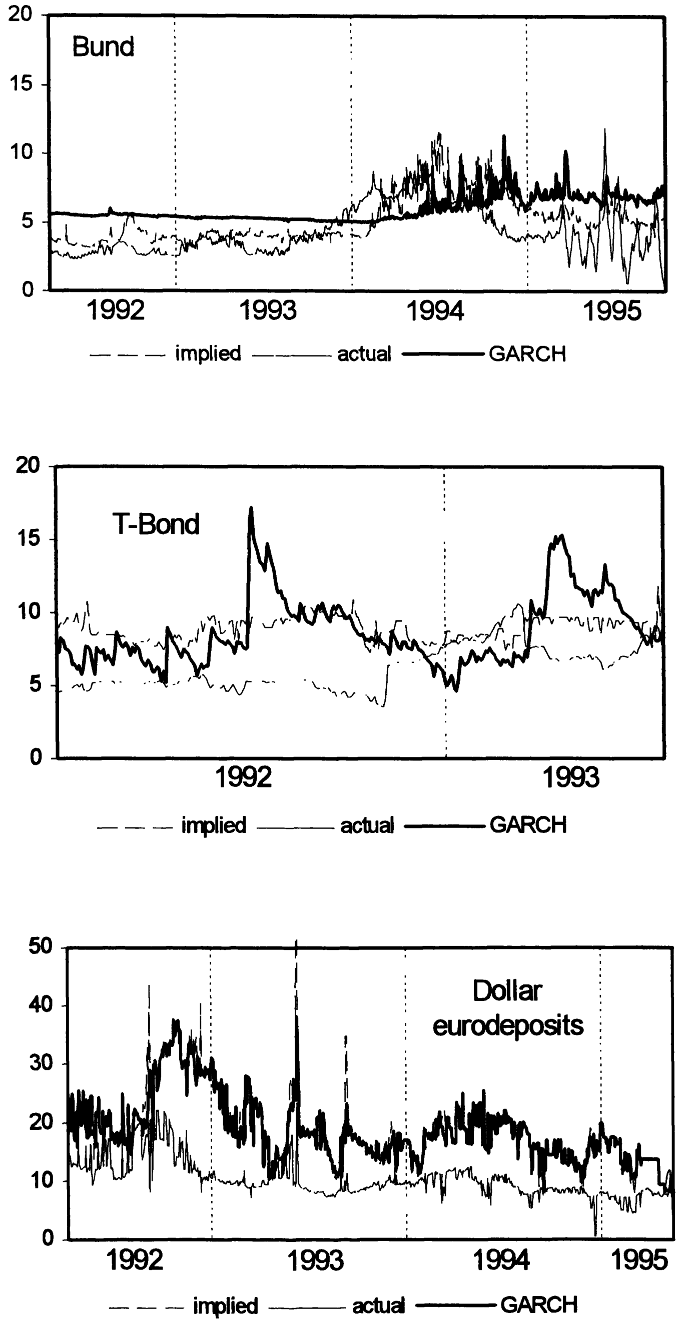
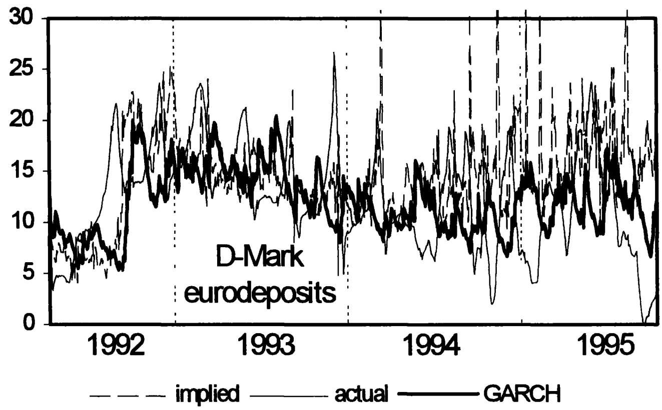
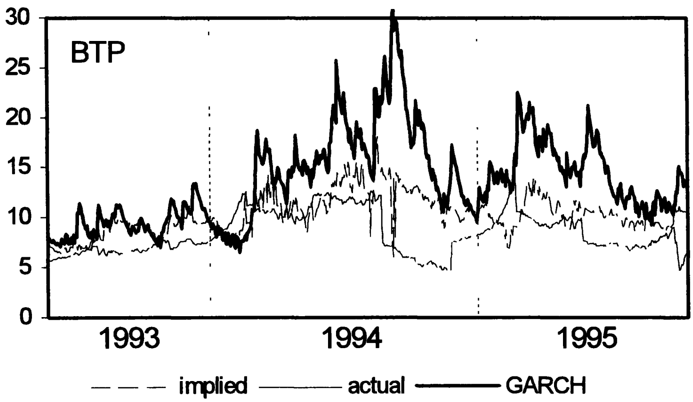


Figure 1b **Measures of variability**





According to **Table 3** the hypothesis that the differences between various volatility measures are equal to zero is not rejected by the appropriate tests, thus implying that neither implied nor GARCH variance are superior predictors of sample variance.

Table 3

Mean absolute difference between measures of variability

(in parentheses standard error)

Comparison between	Bund	T-Bond	BTP	Eurodeposits in D-marks	Eurodeposits in Dollars
Implied an GARCH variance (ε_1)	1.54 (1.20)	1.19 (0.63)	2.38 (2.30)	4.46 (3.42)	3.18 (2.56)
Implied and sample variance (ε_2)	1.38 (1.03)	2.96 (1.51)	2.48 (1.45)	4.71 (3.76)	8.69* (4.76)
GARCH and sample variance (ε_3)	2.52 (1.63)	3.22 (2.25)	4.77 (3,95)	4.39 (3.42)	5.59* (2.92)

*Significant at the 5 percent level of confidence

With the exception of the case of the BTP contract, GARCH estimation referred to the life to maturity of the option provides the best fit for the implicit variance.

The remarkable differences between the various measures of volatility, with none being superior, provide a strong case for monitoring different indicators of variability while adopting a healthy degree of scepticism on the hints suggested by any single measure. At the same time, these results urge for future research aiming at the definition of more general asset pricing models capable of capturing the characteristics of the sample distribution of financial asset returns more accurately. On this score, two avenues appear particularly promising. The first one adopts a parametric approach and originates from the evidence that the distribution of logarithmic rates of change of financial prices are heavy-peaked and tailed compared to a normal distribution (HEYNEN *et al.*, 1994, among the others) and that the autocorrelation function of the absolute logarithmic rates of change in the prices is maximised when the series is raised to a power which differs from two, contrary to the conventional paradigm that the variance provides the most informative measure of variability (FORNARI and MELE, 1996, 1997). As a result, this approach supports the resort to more general statistical distributions, such as the General Error, and the econometric estimation of the power to which the measure of variability has to be raised. However, it implies the drawback that closed form solutions for option prices are typically very difficult to obtain.

The second, non-parametric, approach (BOSSAERTS and HILLION, 1995; GOURIÉROUX *et al.*, 1995; AÏT-SAHALIA, 1996; PATILEA and RENAULT, 1996) does not focus on the randomisation of a single parameter, such as volatility, or on the postulation of more general hypotheses on the conditional distribution of asset returns. Rather, it postulates that the whole risk-neutral probability valuation is stochastic and, in some applications, broadens the set of information relevant to option pricing on the basis of the observed departures of option prices from the predictions of the traditional BLACK and SCHOLES framework.

5. Conclusions

This paper has analysed the information content of the prices of options traded at LIFFE and written on the futures for the Bund, T-Bond and BTP as well as for 3-month eurodeposits denominated in Dollars and Deutsche Marks. First, the informational efficiency of option prices is assessed following the standard procedure of testing whether price changes can be predicted on the basis of their lagged values (weak efficiency) and past observations of price and non-price variables (semi-strong efficiency). The results indicate that, unlike the case of most other financial assets, such regressors indeed have predictive power. This finding, however, should not be seen as implying a verdict of market inefficiency: if the autocorrelation of option prices only reflects the autocorrelation of the variance of the price of the underlying asset (of which there is abundant evidence), no risk-free profits are possible.

The information content of option prices has also been evaluated by comparing the implied measure of volatility with sample variance and with GARCH estimates. Parametric tests show that implied variance does not embody all the information about volatility which can instead be extracted from past behaviour of prices of the underlying asset, even though it adds predictive power to a standard GARCH model – as also does the traded volumes of options, consistently the findings of TAUCHEN and PITTS (1983) and ANDERSEN (1996).

In the short run, major differences can arise between sample variance, implied variance and forecast variance from a GARCH model, even when the latter is obtained through a procedure which ensures consistency with the other measures in terms of available information and relevant time horizon. Sample variance, then, is not necessarily a good proxy of economic agents' perception of volatility, the variable ultimately relevant to their investment decisions. However, over longer periods, the difference between the various measures of volatility is not statistically different from zero, revealing the consistency of economic agents' expectations, as well as of GARCH econometric models, with the actual evolution of the second moments of financial variables.

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