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Simplicity of tangent bundles of smooth horospherical varieties of Picard number one

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Abstract. Recently, Kanemitsu has discovered a counterexample to the long-standing conjecture that the tangent bundle of a Fano manifold of Picard number one is (semi)stable. His counterexample is a smooth horospherical variety. There is a weaker conjecture that the tangent bundle of a Fano manifold of Picard number one is simple.

We prove that this weaker conjecture is valid for smooth horospherical varieties of Picard number one. Our proof follows from the existence of an irreducible family of unbendable rational curves whose tangent vectors span the tangent spaces of the horospherical variety at general points.

Résumé. Récemment, Kanemitsu a découvert un contre-exemple à la conjecture de longue date selon laquelle le faisceau tangent d'une variété de Fano de nombre de Picard un est (semi) stable. Son contre-exemple est une variété horosphérique lisse. Une conjecture plus faible affirme que le fibré tangent d'une variété de Fano de nombre de Picard un est simple.

Nous prouvons que cette conjecture plus faible est valable pour les variétés horosphériques lisses de nombre de Picard un. Notre preuve découle de l'existence d'une famille irréductible de courbes rationnelles non pliables dont les vecteurs tangents engendrent les espaces tangents de la variété horosphérique en des points généraux.

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1. Introduction

Tangent bundles of Fano manifolds have been studied from various points of view, in relation with the existence of a Kähler–Einstein metric. For example, if X admits a Kähler–Einstein metric, then the Lie algebra of holomorphic vector fields on X is reductive. A weaker condition, the existence of a Hermitian–Einstein metric on the tangent bundle TX is equivalent to the polystability of TX , and is equivalent to the stability of TX when X has Picard number one.

In this regard, when X has Picard number one, the tangent bundle TX of a Fano manifold X has been expected to be semistable for a long time.

Conjecture 1 ([9, Conjecture 0.1]). *The tangent bundle of a Fano manifold of Picard number one is semistable.*

This conjecture has been verified in many cases (see e.g., [4, 13]). Recently, however, a counterexample has been discovered by Kanemitsu ([9]): The tangent bundle of a smooth horospherical variety X of Picard number one is not semistable if X is of type $(B_n, \omega_{n-1}, \omega_n)$, where $n \geq 4$ or of type $(F_4, \omega_3, \omega_2)$, and is stable, otherwise. For the types of horospherical varieties of Picard number one, see Proposition 10.

Only scalar multiplications are endomorphisms of a stable vector bundle, so that stable vector bundles are simple. Instead of stability, we consider a weaker conjecture.

Conjecture 2. *The tangent bundle of a Fano manifold of Picard number one is simple.*

In this paper, we prove that Conjecture 2 is valid for any smooth horospherical variety of Picard number one.

Theorem 3. *The tangent bundle of any smooth horospherical varieties of Picard number one is simple.*

To prove Theorem 3 we use unbendable rational curves. A rational curve $f : \mathbb{P}^1 \rightarrow X$ in a uniruled projective manifold X is said to be *unbendable* if the pull-back f^*TX of the tangent bundle TX of X is decomposed as $\mathcal{O}(2) \oplus \mathcal{O}(1)^p \oplus \mathcal{O}^q$ for some nonnegative integers p, q . Minimal rational curves are unbendable but the converse is not true (Remark 8). Here, by a minimal rational curve we mean a rational curve $f : \mathbb{P}^1 \rightarrow X$ with f^*TX being nonnegative, and the degree of f with respect to a fixed ample line bundle is minimal among all such rational curves.

We expect unbendable rational components will play an important role in understanding the geometry of a uniruled projective manifold X as minimal rational components did (see e.g., [6] and [10] and references therein). When X has Picard number one, we get the simplicity of the tangent bundle TX if there is an irreducible family of unbendable rational curves whose tangent vectors span $\mathbb{P}T_x X$ at a general point x in X (Proposition 5).

We prove that for a rational homogeneous variety G/P with G simple or a smooth horospherical variety of Picard number one, such an unbendable rational component exists (Proposition 6 and Proposition 12). Then Theorem 3 follows from Proposition 5 and the condition that X has Picard number one.

In relation with Conjecture 2 we ask whether there is such an unbendable rational component for any Fano manifold of Picard number one.

Conjecture 4. *A Fano manifold of Picard number one admits an irreducible family of unbendable rational curves whose tangent vectors span the tangent spaces of the Fano manifold at general points.*

If Conjecture 4 holds, then so does Conjecture 2 by Proposition 5. Minimal rational curves are unbendable curves, but their tangent vectors do not always generate the tangent space at a general point. Since a Fano manifold of Picard number one is rationally connected, there exists an irreducible family of rational curves whose tangent vectors span the tangent space at a general point, but the restriction of the tangent bundle to rational curves in this family may have more than one $\mathcal{O}(2)$ -factors, or more generally, may have $\mathcal{O}(a)$ -factors with $a \geq 3$. The question is whether we have an irreducible family of rational curves having both properties, rational curves in the family are unbendable and their tangent vectors span the tangent space at a general point.

2. Unbendable rational curves

Let X be a uniruled projective manifold. We say that a rational curve $f : \mathbb{P}^1 \rightarrow X$ in X is

- *unbendable* if the pull-back f^*TX of the tangent bundle TX of X is decomposed as $\mathcal{O}(2) \oplus \mathcal{O}(1)^p \oplus \mathcal{O}^q$ for some nonnegative integers p, q ;

- *minimal* if the pull-back f^*TX of the tangent bundle TX of X is nonnegative, and the degree of f with respect to a fixed ample line bundle is minimal among all such rational curves.

An irreducible component \mathcal{H} of the Hilbert scheme of rational curves in X containing an unbendable (minimal, respectively) rational curve is called an *unbendable (minimal, respectively) rational component* of X .

Fix an unbendable rational component \mathcal{H} of X . For a point $x \in X$ denote by \mathcal{H}_x the subscheme of \mathcal{H} consisting of members passing through x , and define a rational map

$$\tau_x : \mathcal{H}_x \dashrightarrow \mathbb{P}T_xX$$

by sending a rational curve $[C] \in \mathcal{H}_x$ smooth at x to its tangent direction $[T_xC] \in \mathbb{P}T_xX$. The closure \mathcal{C}_x of the image of τ_x is called the *variety of tangents* at x of the family \mathcal{H} .

Proposition 5 ([5, Theorem 2]). *If X has an unbendable rational component \mathcal{H} such that*

- (1) *the variety of tangents at x of \mathcal{H} is nondegenerate in the projective tangent space $\mathbb{P}T_xX$ for general $x \in X$;*
- (2) *a general point of X is joined to a point in X by a connected chain of curves in \mathcal{H} .*

then TX is simple, that is, any endomorphism of TX is a scalar multiplication.

When X has Picard number one, the condition (2) of Proposition 5 is satisfied automatically: There is a sequence of locally closed submanifolds $\mathcal{X}^0 = \{x\} \subsetneq \mathcal{X}^1 \subsetneq \dots \subsetneq \mathcal{X}^m$, where $\dim \mathcal{X}^m = \dim X$, such that any point in \mathcal{X}^k can be connected to a point in \mathcal{X}^{k-1} by a rational curve in \mathcal{H} for any $1 \leq k \leq m$ (see [7, Section 4.3] or [10, Section 3]).

For example, the moduli of semistable vector bundles of rank r and with a fixed determinant ([5]) or a wonderful group compactification ([3]) have an unbendable rational component whose variety of tangents is nondegenerate in $\mathbb{P}(T_xX)$ at a general point $x \in X$. In the first case, in fact, a minimal rational component satisfies the desired property. However, this is not the case in general (Remark 8), and we need to consider an unbendable rational component which is not minimal to get the desired property.

3. Existence of unbendable rational curves

3.1. Rational homogeneous varieties

Let \mathfrak{g} be a complex simple Lie algebra. Let \mathfrak{h} be a Cartan subalgebra and $\Delta = \{\alpha_1, \dots, \alpha_\ell\}$ be a system of simple roots. For each root α , there is an element H_α in \mathfrak{h} satisfying that $\alpha(H) = (H_\alpha, H)$ for any $H \in \mathfrak{h}$, where (\cdot, \cdot) is the Killing form of \mathfrak{g} . Then the Killing form induces a symmetric bilinear form (\cdot, \cdot) on the set Φ of roots defined by $(\alpha, \beta) := (H_\alpha, H_\beta)$ for $\alpha, \beta \in \Phi$. Put $h_\alpha := 2H_\alpha / (\alpha, \alpha)$ and $\alpha^\vee := 2\alpha / (\alpha, \alpha)$ for $\alpha \in \Phi$. Then $\beta(h_\alpha) = (\beta, \alpha^\vee)$.

For a subset $\Delta_1 \subset \Delta$, define a function $n_{\Delta_1} : \Phi \rightarrow \mathbb{Z}$ by

$$n_{\Delta_1}(\alpha) = \sum_{\alpha_i \in \Delta_1} n_i$$

for $\alpha = \sum_{i=1}^\ell n_i \alpha_i \in \Phi$. Define $\Phi_{\Delta_1}^\pm$ and $\Phi_{\Delta_1}^0$ by

$$\Phi_{\Delta_1}^\pm := \{\alpha \in \Phi : \Phi : n_{\Delta_1}(\alpha) \in \mathbb{Z}_\pm\} \quad \text{and} \quad \Phi_{\Delta_1}^0 := \{\alpha \in \Phi : n_{\Delta_1}(\alpha) = 0\}.$$

and put $\mathfrak{p}_{\Delta_1} = \mathfrak{h} \oplus (\oplus_{\alpha \in \Phi_{\Delta_1}^0 \cup \Phi_{\Delta_1}^-} \mathfrak{g}_\alpha)$ and $\mathfrak{m}_{\Delta_1} = \oplus_{\alpha \in \Phi_{\Delta_1}^+} \mathfrak{g}_\alpha$. Then $\Phi = \Phi_{\Delta_1}^- \cup \Phi_{\Delta_1}^0 \cup \Phi_{\Delta_1}^+$ and $\mathfrak{g} = \mathfrak{p}_{\Delta_1} \oplus \mathfrak{m}_{\Delta_1}$. Fix $\mathfrak{p} = \mathfrak{p}_{\Delta_1}$ from now on. Note that \mathfrak{p} contains the negative Borel subalgebra $\mathfrak{b} := \mathfrak{p}_\Delta$.

Let G be a simply connected algebraic group with Lie algebra \mathfrak{g} and P be the subgroup of G with Lie algebra \mathfrak{p} . For each root α , take $E_\alpha \in \mathfrak{g}_\alpha, E_{-\alpha} \in \mathfrak{g}_{-\alpha}$ such that $(E_\alpha, E_{-\alpha}) = 1$. Then

$[E_\alpha, E_{-\alpha}] = H_\alpha \in \mathfrak{h}$ and $E_\alpha, E_{-\alpha}, H_\alpha$ generate a subalgebra \mathfrak{sl}_α of \mathfrak{g} which is isomorphic to \mathfrak{sl}_2 . Let S_α be the subgroup of G with Lie algebra \mathfrak{sl}_α . The S_α -orbit of $o \in G/P$ is a rational curve in G/P , denoted by C_α .

The tangent bundle $T(G/P)$ is the homogeneous vector bundle on G/P associated to the representation $P \rightarrow GL(\mathfrak{g}/\mathfrak{p})$. The tangent bundle $T(G/P)$ restricted to C_α is decomposed as $\bigoplus_{\beta \in \Phi_{\Delta_1}^+} \mathcal{O}(\beta(h_\alpha)) = \bigoplus_{\beta \in \Phi_{\Delta_1}^+} \mathcal{O}((\beta, \alpha^\vee))$ (See [8, proof of Proposition 2]).

Proposition 6. *Let G/P be a rational homogeneous variety. Assume that G is simple. Then there is an unbendable rational component \mathcal{H} whose variety \mathcal{C}_y of tangents at any $y \in G/P$ is nondegenerate in $\mathbb{P}T_y(G/P)$.*

Proof. Let θ be the maximal positive root of \mathfrak{g} . Since θ has nonzero coefficient in α_i for any $i = 1, \dots, \ell$, S_θ is not contained in P_{Δ_1} for any subset Δ_1 of Δ , and thus C_θ is a (nonconstant) rational curve in G/P for any $P = P_{\Delta_1}$. We will show that C_θ is an unbendable rational curve in G/P . It suffices to show that $(\beta, \theta^\vee) \leq 1$ for all $\beta \in \Phi_{\Delta_1}^+ \setminus \{\theta\}$. To do this we will use the following two facts:

- (i) If G is not of type A , then there is $i_0 \in \{1, \dots, \ell\}$ such that $(\alpha_i, \theta) = 0$ for all $i \neq i_0$ in $\{1, \dots, \ell\}$ and $(\alpha_{i_0}, \theta^\vee) = 1$.
- (ii) If G is of type A , then $(\alpha_j, \theta) = 0$ for all $j \in \{2, \dots, \ell - 1\}$ and $(\alpha_1, \theta^\vee) = (\alpha_\ell, \theta^\vee) = 1$.

Following the conventions in [11] for the indices of simple roots, we have $i_0 = 2$ if G is of type B_ℓ, D_ℓ , $i_0 = 1$ if G is of type C_ℓ , and $i_0 = 6$ ($6, 1, 4, 2$, respectively) if G is of type E_6 (E_7, E_8, F_4, G_2 , respectively). See the affine Dynkin diagram of $\Delta^{(1)}$ in [11, Table 6], which is the diagram of the union $\Delta \cup \{-\theta\}$ of Δ in [11, Table 1] and $\{-\theta\}$.

If G is not of type A , then, by (i), the maximal root among roots in $\Phi \setminus \{\theta\}$ is $\theta - \alpha_{i_0}$. Since $(\theta - \alpha_{i_0}, \theta^\vee) = 2 - 1 = 1$, we have $(\beta, \theta^\vee) \leq 1$ for all $\beta \in \Phi_{\Delta_1}^+ \setminus \{\theta\}$. If G is of type A , then, by (ii), among roots in $\Phi \setminus \{\theta\}$, maximal ones are either $\theta - \alpha_1$ or $\theta - \alpha_\ell$. From $(\theta - \alpha_1, \theta^\vee) = (\theta - \alpha_\ell, \theta^\vee) = 1$ it follows that $(\beta, \theta^\vee) \leq 1$ for all $\beta \in \Phi_{\Delta_1}^+ \setminus \{\theta\}$.

The curve C_θ is tangent to $\mathbb{C}E_\theta \subset T_o G/P$. The variety \mathcal{C}_o of tangents at the base point o is the closure of the P -orbit $P.[E_\theta]$ in $\mathbb{P}(\mathfrak{m})$. We claim that \mathcal{C}_o is nondegenerate in $\mathbb{P}(\mathfrak{g}/\mathfrak{p})$. Consider the projection $\mathfrak{g} \rightarrow \mathfrak{g}/\mathfrak{p}$. The closure of the P -orbit $P.[E_\theta]$ in $\mathbb{P}(\mathfrak{g})$ is the G -orbit $G.[E_\theta]$, which is nondegenerate in $\mathbb{P}(\mathfrak{g})$ because it is the highest weight orbit. Therefore, the closure of the P -orbit $P.[E_\theta]$ in $\mathbb{P}(\mathfrak{g}/\mathfrak{p})$ is nondegenerate in $\mathbb{P}(\mathfrak{g}/\mathfrak{p})$. □

Remark 7. The inequality $(\beta, \theta^\vee) \leq 1$ for any $\beta \in \Phi^+ \setminus \{\theta\}$ also follows from [2, Proposition 25 in VI.1.8].

Remark 8. Take a simple root α_i which is not short and consider the rational homogeneous variety G/P associated with $\Delta_1 = \Delta - \{\alpha_i\}$. Then the rational curve C_{α_i} is a minimal rational curve and the variety of tangents of the minimal rational component containing $[C_{\alpha_i}]$ at a point $x \in G/P$ spans a proper subspace of $\mathbb{P}T_x(G/P)$ if the coefficient of α_i in θ is > 1 ([8, Proposition 1]). Therefore, an unbendable rational curve is not necessarily a minimal rational curve.

Together with Proposition 5, Proposition 6 reproves the following result.

Proposition 9 ([1, Theorem 2.1]). *Let $X = G/P$ be a rational homogeneous variety with G being simple. Then the tangent bundle of X is simple.*

Proof. By Proposition 5 and Proposition 6, it suffices to show that a general point x in G/P is joined to the base point o by a connected chain of curves in the unbendable rational component \mathcal{H} constructed in Proposition 6.

Let Σ denote the subset of G/P consisting of $x \in G/P$ which are joined to o by a connected chain of curves in \mathcal{H} . Then the stabilizer Q of Σ in G contains the subgroup of G generated by P and S_θ because the action of S_θ moves the point o to a point in C_θ . Thus Q contains P properly.

Let $\{\omega_1, \dots, \omega_\ell\}$ denote the system of fundamental weights corresponding the system Δ of simple roots. Then each ω_j can be written as a linear combination of α_i 's with positive coefficients (See [11, Table 2]). By (i) and (ii) in the proof of Proposition 6, (ω_j, θ^\vee) is positive for any $j = 1, \dots, \ell$. Therefore, C_θ cannot be contained in a fiber of a nontrivial projection $G/P \rightarrow G/Q$. Since Q/P is not a point, G/Q is a point, that is, G is Q and Σ is G/P . \square

3.2. Smooth horospherical varieties of Picard number one

For $i = 1, \dots, \ell$, let V_{ω_i} be the irreducible representation of G of highest weight ω_i and let v_{ω_i} be a highest weight vector of V_{ω_i} . Let (G, ω_i, ω_j) denote the closure of the G -orbit of the sum $[v_{\omega_i} + v_{\omega_j}]$ in $\mathbb{P}(V_{\omega_i} \oplus V_{\omega_j})$.

Proposition 10 ([12]). *Let X be a smooth horospherical variety of Picard number one. Then, X is either a rational homogeneous variety or one of the following:*

- (1) $(B_n, \omega_{n-1}, \omega_n)$ ($n \geq 3$)
- (2) $(B_3, \omega_1, \omega_3)$
- (3) $(C_n, \omega_k, \omega_{k-1})$ ($n \geq 2, 2 \leq k \leq n$)
- (4) $(F_4, \omega_3, \omega_2)$
- (5) $(G_2, \omega_2, \omega_1)$.

In the latter case, the automorphism group $\text{Aut}^0(X)$ is given by $\tilde{G} \times H^0(G/P, G \times_P V)$, where \tilde{G} is a reductive group with G a maximal semisimple subgroup, and the open $\text{Aut}^0(X)$ -orbit \mathcal{O} in X is G -equivariantly isomorphic to the total space of a homogeneous vector bundle $G \times_P V$ on G/P , where P is the maximal parabolic subgroup associated to

$$\omega_{n-1}, \omega_1, \omega_{k-1}, \omega_3, \omega_2, \text{ respectively}$$

and V is the simple P -module of highest weight λ_V given by

$$\omega_{n-1} - \omega_n, \omega_1 - \omega_3, \omega_k - \omega_{k-1}, \omega_3 - \omega_2, \omega_2 - \omega_1, \text{ respectively.}$$

Remark 11. We remark that our convention is different from that of [12]. The way of indexing simple roots and fundamental weights is the same as in [11] in this paper while [12] follows the convention in [2]. The isotropy subgroup P contains the negative Borel subgroup in this paper while it contains the positive Borel subgroup in [12].

Proposition 12. *Let X be a smooth horospherical variety of Picard number one. Then there is an unbendable rational component \mathcal{H} whose variety \mathcal{C}_x of tangents at any x in the open $\text{Aut}^0(X)$ -orbit \mathcal{O} is nondegenerate in $\mathbb{P}T_x X$.*

Proof. We will show that there is an unbendable rational component \mathcal{H} such that for any point x in the open $\text{Aut}^0(X)$ -orbit \mathcal{O} , the variety of tangents of \mathcal{H} at x is nondegenerate in $\mathbb{P}T_x X$. Since $G \times H^0(G/P, G \times_P V)$ acts on $\mathcal{O} = G \times_P V$ transitively, we may assume that $x = [o, 0]$, where o is the base point in G/P and 0 is the zero element in V . Identify G/P with the zero section Y of $G \times_P V$ and let C_θ be the rational curve in $G/P \simeq Y$ constructed in Proposition 6. We claim that C_θ is an unbendable curve in \mathcal{O} .

Let λ_V be the highest weight of V listed in Proposition 10. By the facts (i) and (ii) in the proof of Proposition 6, (λ_V, θ^\vee) is the coefficient of α_{i_0} in the expression of λ_V via simple roots, $\alpha_1, \dots, \alpha_\ell$. For the description of ω_i as a linear combination of simple roots, see [11, Table 2]: The i -th column of the matrix $(A^t)^{-1}$ inverse to the transposed Cartan matrix A is the list of coefficients of simple roots in ω_i . Let $b_{i,j}$ denote the (i, j) -th element of the matrix $(A^t)^{-1}$. Then (λ_V, θ^\vee) is given by

$$b_{2,n-1} - b_{2,n}, \quad b_{2,1} - b_{2,3}, \quad b_{1,k} - b_{1,k-1}, \quad b_{4,3} - b_{4,2}, \quad b_{2,2} - b_{2,1}, \quad \text{ respectively.}$$

Using the description of the matrix $(A^t)^{-1} = (b_{i,j})$ in [11, Table 2] we get that (λ_V, θ^\vee) is given by

$$\frac{1}{2}(4-2), \quad \frac{1}{2}(2-2), \quad \frac{1}{2}(4-4), \quad 3-2, \quad 2-1, \quad \text{respectively.}$$

Therefore, (λ_V, θ^\vee) is 1 or 0, and thus the vector bundle $G \times_P V$ restricted to $C_\theta \subset G/P$ splits as $\oplus_{i=1}^m \mathcal{O}(a_i)$, where

$$1 \geq a_1 \geq \cdots \geq a_m \geq 0.$$

From this property and the nonnegativity of $TY|_{C_\theta}$, it follows that the short exact sequence $0 \rightarrow TY \rightarrow TX|_Y \rightarrow G \times_P V \rightarrow 0$ restricted to C_θ splits. Since $TY|_{C_\theta}$ has only one $\mathcal{O}(2)$ -factor by Proposition 6, we get that $TX|_{C_\theta}$ is also nonnegative and has only one $\mathcal{O}(2)$ -factor.

Now let \mathcal{V} be the vector space spanned by

$$\{T_{[o,0]}(aC_\theta) : [o,0] \in aC_\theta, a \in G \times H^0(G/P, G \times_P V)\}.$$

Then the projection of \mathcal{V} to V is a nonzero P -stable subspace of V , and thus is the whole V because V is an irreducible P -module. Hence the tangent directions to translates of C_θ passing through $[o,0]$ span $T_{[o,0]}\mathcal{O}$. \square

Proof of Theorem 3. By Proposition 12, there is an unbendable rational component \mathcal{H} whose variety of tangents is nondegenerate in $\mathbb{P}(T_x X)$ for any $x \in \mathcal{O}$. Since X has Picard number one, by Proposition 5, any endomorphism of the tangent bundle of X is a scalar multiple. \square

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