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Ordinary differential equations/Partial differential equations

Blow-up solutions for a general class of the second-order differential equations on the half line

Solutions non bornées d'une classe générale d'équations différentielles du second ordre sur la demi-droite

Angelo R.F. de Holanda

Universidade Federal de Campina Grande, Unidade Acadêmica de Matemática, CEP:58109-970, Campina Grande - PB, Brazil

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ABSTRACT

In this paper, we study the existence of positive blow-up solutions for a general class of the second-order differential equations and systems, which are positive radially symmetric solutions to many elliptic problems in $\mathbb{R}^N$. We explore fixed point arguments applied to suitable integral equations to get solutions.

RÉSUMÉ

Nous étudions dans ce texte l'existence de solutions positives, non bornées, pour une classe générale d'équations et systèmes différentiels du second ordre. Il s'agit de solutions à symétrie radiale, positives, de maints problèmes elliptiques dans $\mathbb{R}^N$. Pour obtenir ces solutions, nous passons par des arguments de point fixe pour des opérateurs intégraux adéquats.

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1. Introduction

In this short paper, we investigate the existence of positive blow-up solutions for the following class of second-order differential equations

$$(r^{\alpha}|u'(r)|^{\beta}u'(r))' = \lambda r^{\gamma} f(r, u(r), |u'(r)|), \quad r > 0, \tag{P}$$

and for the system

$$\begin{cases} (r^{\alpha_1}|u'(r)|^{\beta_1}u'(r))' = \lambda r^{\gamma_1} f_1(r, u(r), v(r), |u'(r)|, |v'(r)|), & r > 0, \\ (r^{\alpha_2}|v'(r)|^{\beta_2}v'(r))' = \mu r^{\gamma_2} f_2(r, u(r), v(r), |u'(r)|, |v'(r)|), & r > 0, \end{cases} \tag{S}$$

E-mail address: angelo@mat.ufcg.edu.br.

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where λ and μ are real parameters, α, α_i, β, β_i, γ and γ_i $(i=1,2)$ are real constants. The functions $f:\overline{\mathbb{R}}_+\times\mathbb{R}_+\times\overline{\mathbb{R}}_+\longrightarrow\overline{\mathbb{R}}_+$, $f_i:\overline{\mathbb{R}}_+\times\mathbb{R}_+\times\mathbb{R}_+\times\overline{\mathbb{R}}_+\times\overline{\mathbb{R}}_+\longrightarrow\overline{\mathbb{R}}_+$, $(i=1,2)$ are positive continuous and $\mathbb{R}_+=(0,\infty)$, $\overline{\mathbb{R}}_+=[0,\infty)$.

A function $u:\overline{\mathbb{R}_+}\longrightarrow\mathbb{R}_+$ is said to be a solution to problem (P) if

$$r^{\alpha}|u'(r)|^{\beta}u'(r)\in C^1(\overline{\mathbb{R}}_+),\ \lim_{r\to 0} r^{\alpha}|u'(r)|^{\beta}u'(r)=0 \text{ if } \alpha<0,$$

and u satisfies equation (P) and similarly the pair $(u,v):\overline{\mathbb{R}_+}\longrightarrow\mathbb{R}_+$ is a solution to the system (S) if

$$r^{\alpha_1}|u'(r)|^{\beta_1}u'(r), r^{\alpha_2}|v'(r)|^{\beta_2}v'(r)\in C^1(\overline{\mathbb{R}}_+),$$
$$\lim_{r\to 0} r^{\alpha_1}|u'(r)|^{\beta_1}u'(r)=\lim_{r\to 0} r^{\alpha_2}|v'(r)|^{\beta_2}v'(r)=0 \text{ if } \alpha_i<0,\ i=1,2,$$

and (u,v) satisfies the system (S).

The problems (P) and (S) are models of many elliptic problems when we are looking for radially symmetric solutions. For instance, if $\alpha=N-k$, $\beta=k-1$ and $\gamma=N-1$ with $k=1,2,\ldots,N$, then positive blow-up solutions to (P) are solutions to the problem

$$\begin{cases} S_k(D^2u)=\lambda C(N,k)f(|x|,u,|\nabla u|) \text{ in } \mathbb{R}^N,\\ u>0 \text{ in } \mathbb{R}^N,\\ u\longrightarrow\infty \quad\text{as}\quad |x|\longrightarrow\infty, \end{cases} \tag{1.1}$$

where $S_k(D^2u)$ is the *k-Hessian Operator* and $C(N,k)=(N-1)!/(N-k)!(k-1)!k$, which includes as a special case the *Monge–Ampère Operator* when $k=N$.

The second problem studied is related to the following system,

$$\begin{cases} \operatorname{div}(a_1(x)|\nabla u|^{\beta_1}\nabla u)=\lambda b_1(x)f_1(|x|,u,v,|\nabla u|,|\nabla v|) \text{ in } \mathbb{R}^N,\\ \operatorname{div}(a_2(x)|\nabla v|^{\beta_2}\nabla v)=\mu b_2(x)f_2(|x|,u,v,|\nabla u|,|\nabla v|) \text{ in } \mathbb{R}^N,\\ u,v>0 \text{ in } \mathbb{R}^N,\\ u,v\longrightarrow\infty \quad\text{as}\quad |x|\longrightarrow\infty, \end{cases} \tag{1.2}$$

where $a_i(x)=|x|^{k_i}$, $b_i(x)=|x|^{l_i}$ with $k_i,l_i\in\mathbb{R}$ and $\alpha_i=k_i+N-1$, $\gamma_i=l_i+N-1$ $(i=1,2)$. Here we would like to attract the reader's attention to the fact that functions a_i and b_i may be singular at the origin. When $k_i=l_i=0$ and $\beta_1=p-2$, $\beta_2=q-2$ with $1<p,q<\infty$, we obtain a *(p,q)-Laplacian System*.

Besides the problems (1.1), (1.2) and their versions for system and simple equation, respectively, we believe that our methods also can be applied with slight modifications of (P) and (S) to establish the existence of large solutions defined on the whole space $\mathbb{R}^N$ for other classes of quasilinear problems like

$$\Delta_{\phi}u=\lambda f(|x|,u,|\nabla u|) \text{ in } \mathbb{R}^N,$$

and systems

$$\begin{cases} \Delta_{\phi_1}u=\lambda f_1(|x|,u,v,|\nabla u|,|\nabla v|) \text{ in } \mathbb{R}^N,\\ \Delta_{\phi_2}v=\mu f_2(|x|,u,v,|\nabla u|,|\nabla v|) \text{ in } \mathbb{R}^N, \end{cases}$$

where $\Delta_{\phi}u:=\operatorname{div}(\phi(|\nabla u|)\nabla u)$, is the ϕ-Laplacian operator (see, Fukagai and Narukawa [2]).

The problems (P) and (S) considered here were motivated by Covei [1], Zhang and Zhou [5] (see also Zhang [4]); these authors discussed the case in which the right-hand side does not depend on the gradient ∇u. Furthermore, we are able to treat more general classes of quasilinear equations.

Now, inspired by Kuzano and Swanson [3], we assume the following hypotheses.

First of all, we consider $i\in\{0,1,2\}$, $\alpha_i^+:=\max(\alpha_i,0)$, $\alpha_i^-:=\max(0,-\alpha_i)$ and define the auxiliary function

$$\phi_i(t):=\frac{\beta_i+1}{\alpha_i^-+\beta_i+1}t^{(\alpha_i^-+\beta_i+1)/(\beta_i+1)},\ t\ge 0,$$

where $\alpha_0=\alpha$, $\beta_0=\beta$, $\gamma_0=\gamma$ and $\phi_0=\phi$.

(h1) Suppose that $\beta_i>-1$ and $\gamma_i\ge\alpha_i^+$.
(h2) $f(t,x,z)$ is a nondecreasing function with respect to x and z for a fixed values of the other variables. That is, the functions $x\mapsto f(t,x,z)$ and $z\mapsto f(t,x,z)$ are nondecreasing, for fixed (t,z) and fixed (t,x) respectively.
(h3) $f_i(t,x,y,z,w)$, $i=1,2$ is a nondecreasing function with respect to x, y, z and w for fixed values of the other variables.
(h4) There exists a constant $\delta>0$ such that

$$\int_0^{\infty} t^{\gamma-\alpha^+}f(t,\delta(\phi(t)+1),\delta\phi'(t))\,dt<\infty.$$

(h5) There exist constants $\delta_i > 0$ such that

$$\int_0^\infty t^{\gamma_i-\alpha_i^+} f_i(t, \delta_1(\phi_1(t)+1), \delta_2(\phi_2(t)+1), \delta_1\phi_1'(t), \delta_2\phi_2'(t))\,dt < \infty,\ i = 1, 2.$$

Our aim is to find increasing solutions u of (P) and increasing solutions (u, v) of (S) respectively subject to the initial conditions $u(0) = \xi$ and $(u(0), v(0)) = (\xi_1, \xi_2)$, for some positive values of ξ, ξ_1 and ξ_2. Thus, any solution u to (P) is a function such that $u'(r) > 0$ if $r > 0$ and satisfies the integral equation

$$u(r) = \xi + \int_0^r \left(\lambda s^{-\alpha} \int_0^s t^\gamma f(t, u(t), u'(t))\,dt\right)^{1/\beta+1} ds,\ r > 0.$$

Similarly, any solution (u, v) of (S) is such that u and v are increasing functions of the variable r and satisfies the following system of integral equations

$$\begin{cases} u(r) = \xi_1 + \int_0^r \left(\lambda s^{-\alpha_1} \int_0^s t^{\gamma_1} f_1(t, u(t), v(t), u'(t), v'(t))\,dt\right)^{1/\beta_1+1} ds,\ r > 0, \\ v(r) = \xi_2 + \int_0^r \left(\mu s^{-\alpha_2} \int_0^s t^{\gamma_2} f_2(t, u(t), v(t), u'(t), v'(t))\,dt\right)^{1/\beta_2+1} ds,\ r > 0. \end{cases} \tag{1.3}$$

Our main results are as follows.

Theorem 1.1. *Assume* $(h1)$ *with* $i = 0$, $(h2)$ *and* $(h4)$. *Then there is* $\Lambda > 0$ *such that for each* $\lambda \in (0, \Lambda]$ *and for each* $\xi \in (0, \delta]$, *problem* (P) *admits increasing solution* u *satisfying*

$$\xi \le u(r) \le \xi(\phi(r)+1),\ r \ge 0,$$
$$u(r) \longrightarrow \infty \quad \text{as} \quad r \longrightarrow \infty,$$

and u *is strictly convex if* $\alpha \le 0$.

Theorem 1.2. *Assume* $(h1)$ *with* $i = 1, 2$, $(h3)$ *and* $(h5)$. *Then there is* $\Lambda > 0$ *such that for each* $\lambda, \mu \in (0, \Lambda]$ *and for each* $\xi_1, \xi_2 \in (0, \delta]$, *problem* (S) *admits increasing solution* (u, v) *satisfying*

$$\xi_1 \le u(r) \le \xi_1(\phi_1(r)+1), \quad \xi_2 \le v(r) \le \xi_2(\phi_2(r)+1), \quad r \ge 0,$$
$$u(r) \longrightarrow \infty, \quad v(r) \longrightarrow \infty \quad \text{as} \quad r \longrightarrow \infty,$$

and (u, v) *is strictly convex if* $\alpha_1, \alpha_2 \le 0$.

2. Proof of Theorem 1.1

For a fixed choice of ξ in the interval $(0, \delta]$ and λ a small positive parameter, the solutions to (P) are fixed points of the compact operator

$$(Fu)(r) = \xi + \int_0^r \left(\lambda s^{-\alpha} \int_0^s t^\gamma f(t, u(t), u'(t))\,dt\right)^{1/\beta+1} ds$$

on the closed convex set

$$\mathcal{C} = \left\{u \in C^1(\overline{\mathbb{R}}_+);\ u(0) = \xi,\ 0 \le u'(r) \le \xi\phi'(r),\ r \ge 0\right\}.$$

In order to prove that $F : \mathcal{C} \longrightarrow C^1(\overline{\mathbb{R}}_+)$ has a fixed point, we need consider a countable family of semi-norms

$$p_n(u) := \max_{r \in I_n} \left\{|u(r)|, |u'(r)|\right\},\ I_n := [0, n], n = 1, 2, 3, \ldots$$

and the invariant complete metric

$$d(u,u_0)=\sum_{n=1}^{\infty}\frac{2^{-n}p_n(u-u_0)}{1+p_n(u-u_0)};\ u,u_0\in C^1(\overline{\mathbb{R}}_+), \tag{2.1}$$

under which the space $C^1(\overline{\mathbb{R}}_+)$ becomes a Fréchet space. For simplicity, we denote this topology by C^1, that is $C^1:=\big(C^1(\overline{\mathbb{R}}_+),d\big)$.

Preliminary results

Now we present some auxiliary results that will be used. The first lemma is a well-known inequality, which is obtained from Mean Value Theorem, whose proof we omit.

Lemma 2.1. *Let $\sigma>-1$. There exists a constant $C_\sigma>0$ such that*

$$||x|^\sigma x-|y|^\sigma y|\le C_\sigma(|x|^\sigma+|y|^\sigma)|x-y|, \tag{2.2}$$

for all $x,\ y\in\mathbb{R}$.

Lemma 2.2. *Suppose $(h1)$ with $i=0$, $(h2)$ and $(h4)$ and let $\xi\in(0,\delta]$. There exists $\Lambda>0$ such that if $\lambda\in(0,\Lambda]$ then $F\mathcal{C}\subset\mathcal{C}$.*

Proof. Assume that ξ is fixed such that $0<\xi\le\delta$. For any $u\in\mathcal{C}$ we have $(Fu)(0)=\xi$. From (h_2) and (h_4), there is $M>0$ such that

$$\lambda\int_0^\infty t^{\gamma-\alpha^+}f(t,\xi(\phi(t)+1),\xi\phi'(t))\,\mathrm{d}t\le\lambda M,\ \lambda>0.$$

Hence, there exist $\Lambda>0$ such that

$$\lambda\int_0^\infty t^{\gamma-\alpha^+}f(t,\xi(\phi(t)+1),\xi\phi'(t))\,\mathrm{d}t\le\xi^{\beta+1},$$

for all $\lambda\in(0,\Lambda]$. Since $u\in\mathcal{C}$, then $u(r)\le\xi(\phi(r)+1),r>0$. It follows from (h_4) that

$$\begin{aligned}0\le(Fu)'(r)\ &=\ \left(\lambda r^{-\alpha}\int_0^r t^\gamma f(t,u(t),u'(t))\,\mathrm{d}t\right)^{1/\beta+1}\\ &\le\ r^{\frac{\alpha^-}{\beta+1}}\left(\lambda\int_0^\infty t^{\gamma-\alpha^+}f(t,\xi(\phi(t)+1),\xi\phi'(t))\,\mathrm{d}t\right)^{1/\beta+1}\\ &\le\ \xi\phi'(r),\quad r\ge0.\end{aligned}$$

From the above analysis, $F\mathcal{C}\subset\mathcal{C}$. □

Lemma 2.3. *$F:\mathcal{C}\longrightarrow C^1(\overline{\mathbb{R}}_+)$ is continuous in the C^1-topology.*

Proof. Let $\{u_j\}\subset\mathcal{C}$ and $u\in\mathcal{C}$ such that $d(u_j,u)\longrightarrow0$. It follows that $f(r,u_j,u_j')\longrightarrow f(r,u,u')$ uniformly on I_n. Then, given any $\epsilon>0$, there is a positive integer $J_0=J_0(\epsilon,n)$ such that

$$|r^{\gamma-\alpha^+}\{f(r,u_j,u_j')-f(r,u,u')\}|<\frac{\epsilon}{n^{1+\alpha^-}}\ ,\ \ j\ge J_0,\ r\in I_n.$$

Therefore

$$|r^{-\alpha}\int_0^r t^\gamma f(r,u_j,u_j')\,\mathrm{d}t-r^{-\alpha}\int_0^r t^\gamma f(r,u,u')\,\mathrm{d}t|\le r^{\alpha^-}\int_0^r|t^{\gamma-\alpha^+}\{f(r,u_j,u_j')-f(r,u,u')\}|\,\mathrm{d}t<\epsilon$$

for $j\ge J_0$ and $r\in I_n$. Then we get the convergences $(Fu_j)'\longrightarrow(Fu)'$, $(Fu_j)\longrightarrow(Fu)$ uniformly on I_n. Thus $p_n(Fu_j-Fu)\longrightarrow0$, which implies that $d(Fu_j,F_u)\longrightarrow0$. So F is continuous. □

Lemma 2.4. *$F\mathcal{C}\subset\mathcal{C}$ is relatively compact in the C^1-topology.*

Proof. Set $[F\mathcal{C}]=\{Fu/u\in\mathcal{C}\}$, $[F\mathcal{C}]'=\{(Fu)'/u\in\mathcal{C}\}$ and note that they are uniformly bounded. Now, we show $[F\mathcal{C}]$ and $[F\mathcal{C}]'$ is locally equicontinuous. Let $r_1, r_2\in[a,b]$ with $0\le r_1<r_2$. Thus

$$|(Fu)(r_2)-(Fu)(r_1)|\le\xi(\phi(r_2)-\phi(r_1))$$

from which $[F\mathcal{C}]$ is equicontinuous in any $[a,b]\subset\overline{\mathbb{R}}_+$. On the other hand,

$$|(Fu)'(r_2)-(Fu)'(r_1)|=|(r_2^{-\alpha}\int_0^{r_2}t^\gamma f(t,u,u')\,\mathrm{d}t)^{\frac{1}{\beta+1}}-(r_1^{-\alpha}\int_0^{r_1}t^\gamma f(t,u,u')\,\mathrm{d}t)^{\frac{1}{\beta+1}}|.$$

From inequality (2.2) with $\sigma+1=\frac{1}{\beta+1}$,

$$\begin{aligned}|(Fu)'(r_2)-(Fu)'(r_1)| \le\; & C_\sigma|(r_2^{-\alpha}\int_0^{r_2}t^\gamma f(t,u,u')\,\mathrm{d}t)^\sigma+(r_1^{-\alpha}\int_0^{r_1}t^\gamma f(t,u,u')\,\mathrm{d}t)^\sigma| \\ & |(r_2^{-\alpha}\int_0^{r_2}t^\gamma f(t,u,u')\,\mathrm{d}t)-(r_1^{-\alpha}\int_0^{r_1}t^\gamma f(t,u,u')\,\mathrm{d}t)|.\end{aligned}$$

Next, we will divide our study into two cases:

Case 1: $\sigma\ge0$ (that is $-1<\beta\le0$).

Since $u\in\mathcal{C}$ and $\xi\in(0,\delta]$ it follows that

$$|(Fu)'(r_2)-(Fu)'(r_1)|\le C_\sigma|(r_2^{-\alpha}\int_0^{r_2}t^\gamma f(t,u,u')\,\mathrm{d}t)-(r_1^{-\alpha}\int_0^{r_1}t^\gamma f(t,u,u')\,\mathrm{d}t)|$$

for all $r_1, r_2\in[a,b]$. Consider the function $g(r):=r^{-\alpha}\int_0^r t^\gamma f(t,u,u')\,\mathrm{d}t$ and notice that

$$|g'(r)|\le(1+|\alpha|)b^{\gamma-\alpha}f(r,\delta(\phi(r)+1),\delta\phi'(r))),\ r\in[a,b].$$

Thus, $\sup_{r\in[a,b]}|g'(r)|<\infty$ and by the Mean Value Theorem,

$$|(Fu)'(r_2)-(Fu)'(r_1)|\le C_\sigma|r_2-r_1|\ \text{ with }\ r_1,r_2\in[a,b].$$

Implying the equicontinuity of $[F\mathcal{C}]'$ on any $[a,b]\subset\overline{\mathbb{R}}_+$.

Case 2: $-1<\sigma<0$ (that is $\beta>0$).

Now, if $[a,b]\subset\overline{\mathbb{R}}_+$ with $0<a_0\le a$, then we proceed as in the Case 1. On the other hand, if $a=0$, since the function $h:[0,1]\longrightarrow\mathbb{R};\ h(g)=g^{\sigma+1}$, $0<\sigma+1<1$ is Hölder continuous and $\lim_{r\to0}g(r)=0$, we have

$$|(Fu)'(r_2)-(Fu)'(r_1)|=|(g(r_2))^{\sigma+1}-(g(r_1))^{\sigma+1}|\le C|g(r_2)-g(r_1)|^{\sigma+1}$$

and once again, by the Mean Value Theorem we obtain the equicontinuity of $[F\mathcal{C}]'$ on any $[0,b]$.

Therefore, from Ascoli's Theorem, it follows that $[F\mathcal{C}]$ and $[F\mathcal{C}]'$ are relatively compact in any compact interval of $\overline{\mathbb{R}}_+$. Consequently, by a diagonal argument, we conclude that $F\mathcal{C}$ is relatively compact in the C^1-topology. □

Proof of Theorem 1.1 (closing)

By Lemma 2.2, we have $\overline{F\mathcal{C}}\subset\overline{\mathcal{C}}=\mathcal{C}$, thus $\widetilde{\mathcal{C}}:=\overline{conv}(\overline{F\mathcal{C}})\subset\mathcal{C}$ is compact and $F\widetilde{\mathcal{C}}\subset\widetilde{\mathcal{C}}$. Therefore, the Schauder–Tychonov fixed point theorem implies that $F|\widetilde{\mathcal{C}}:\widetilde{\mathcal{C}}\longrightarrow\widetilde{\mathcal{C}}$ has a fixed point. Then, there is $u\in\mathcal{C}$ such that

$$u(r)=\xi+\int_0^r\left(\lambda s^{-\alpha}\int_0^s t^\gamma f(t,u(t),u'(t))\,\mathrm{d}t\right)^{1/\beta+1}\mathrm{d}s.$$

It follows that $u'(r)\ge0$, $r\ge0$, $u'(0)=0$ and

$$\xi\le u(r)\le\xi(\phi(r)+1),\ r\ge0.$$

Furthermore, u satisfies

$$((u'(r))^{\beta+1})'+\frac{\alpha}{r}(u'(r))^{\beta+1}=\lambda r^{\gamma-\alpha}f(r,u,u'),\ r\ge0.$$

If $\alpha \le 0$, we have $u''(r) > 0$ for all $r > 0$ since

$$(\beta+1)(u'(r))^{\beta+1}u''(r) \ge \lambda r^{\gamma-\alpha} f(r,u,u') > 0,\ r>0,$$

from where it follows that u is strictly convex. On the other hand, given $\alpha > 0$ we obtain $r_0 > 0$ such that

$$\alpha < \frac{\lambda r_0^{\gamma-\alpha+1}}{\xi^{\beta+1}} \inf_{r\in[r_0,\infty)} f(r,\xi,0),$$

and again $u''(r) > 0$ for all $r > r_0$. Hence,

$$u(r) \ge u'(r_0)(r-r_0) \longrightarrow \infty \text{ if } r \longrightarrow \infty. \qquad \square$$

3. Proof of Theorem 1.2

To find solutions to the system (S), consider the mapping

$$I(u,v) = \big(F_1(u,v), F_2(u,v)\big)$$

defined by the system of integral equations

$$F_1(u,v)(r) = \xi_1 + \int_0^r \left(\lambda s^{-\alpha_1} \int_0^s t^{\gamma_1} f_1(t,u(t),v(t),u'(t),v'(t))\,\mathrm{d}t\right)^{1/\beta_1+1} \mathrm{d}s,$$

$$F_2(u,v)(r) = \xi_2 + \int_0^r \left(\mu s^{-\alpha_2} \int_0^s t^{\gamma_2} f_2(t,u(t),v(t),u'(t),v'(t))\,\mathrm{d}t\right)^{1/\beta_2+1} \mathrm{d}s,$$

on the closed convex set $\mathcal{C} := \mathcal{C}_1 \times \mathcal{C}_2$, where

$$\mathcal{C}_i = \left\{u \in C^1(\overline{\mathbb{R}}_+);\ u(0)=\xi_i,\ 0 \le u'(r) \le \phi_i'(r),\ r \ge 0\right\},\ i=1,2.$$

Similar to what was done previously, $I : \mathcal{C} \longrightarrow C^1 \times C^1$ is well defined, is continuous, and $I\mathcal{C} \subset \mathcal{C}$ is relatively compact in the C^1-topology by considering the metric $d'((u,u_0),(v,v_0)) = d(u,u_0) + d(v,v_0)$ where d is given by (2.1). We can then apply the Schauder–Tychonov fixed point theorem to conclude that there exists an element $(u,v) \in \mathcal{C}$ such that $I(u,v) = (u,v)$. Thus (u,v) satisfies (1.3), and hence also (u,v) is a solution to the original system (S). To finish the proof, we proceed as in the proof of the Theorem 1.1. $\square$

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