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Localization of extended current source with finite frequencies

Localisation de sources étendues à partir d'un nombre fini de fréquences

Abdul Wahab [a], Amer Rasheed [a], Rab Nawaz [b], Saman Anjum [a]

[a] *COMSATS Institute of Information Technology, G. T. Road, 47040, Wah Cantt., Pakistan*
[b] *COMSATS Institute of Information Technology, Park Road, Chak Shahzad, 44000, Islamabad, Pakistan*

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A B S T R A C T

A phase conjugation algorithm for localizing the spatial support of an extended radiating current source from boundary measurements of the electric field over a finite set of frequencies is presented. An imaging function using a full frequency bandwidth is established and analyzed. It is subsequently adopted to the case of finite frequency measurements. Finally, the algorithm is blended with l_1-regularization in order to deal with the artifacts associated with finite frequency data.

R É S U M É

Dans cette note, nous présentons un algorithme de conjugaison de phase pour la reconstruction d'une source étendue à partir de mesures de champ électrique obtenues pour un ensemble fini de fréquences. Nous commençons par introduire et analyser une fonctionnelle d'imagerie à partir de mesures obtenues pour un intervalle de fréquences. Ensuite, nous proposons une régularisation l_1 d'une telle fonctionnelle d'imagerie afin d'éliminer les artefacts dus à l'aspect discret et limité des fréquences utilisées.

1. Introduction

Inverse source problems have been the subject of numerous studies over the recent past due to a plethora of applications in science and engineering, especially in biomedical imaging, non-destructive testing and prospecting geophysics (see, e.g., [1,2,5,3,4]). Several frameworks to recover spatial and temporal support of the acoustic, elastic and electromagnetic sources in time and frequency domains have been developed [3,4,13,14], including time reversal and phase conjugation algorithms [8,10–12].

The simplicity and robustness of time reversal and phase conjugation algorithms motivate their application to dealing with source localization problems. If the sources are spatially punctual (Dirac delta sources), a single measurement of the emitted wave over an imaging surface on single time or frequency is sufficient to locate the source position. However, when the sources are extended in space, the problem is well-posed when the available data are collected over an interval of time

E-mail addresses: wahab@ciitwah.edu.pk (A. Wahab), amerasheed@ciitwah.edu.pk (A. Rasheed), rabnawaz@comsats.edu.pk (R. Nawaz), shum.88@hotmail.com (S. Anjum).

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$[0, T]$ for sufficiently large $T > 0$ or over a large frequency bandwidth. The data collected at each time or frequency contain useful information about an extended source that can be retrieved on re-emission of the captured wave after time reversal or phase conjugation. The superposition of all the retrieved information provides the support of extended sources with a resolution limited by Rayleigh's diffraction criterion [2,5].

In real physical configurations, the only measurements available are made over a finite set of frequencies or time instances. The lack of complete information induces noise in the reconstruction of the extended source. All one can get is an initial guess of the support of a source.

This work aims at recovering a radiating electromagnetic source using boundary measurements of the electric field over a finite set of frequencies using a phase conjugation algorithm blended with *fast iterative shrinkage thresholding algorithm with backtracking* of Beck and Teboulle [7] for l_1-regularization. First a phase conjugation imaging function using a full frequency bandwidth is established to recover the spatial support of the source. Then it is adopted to the case of a finite number of frequencies. If broadband frequency information is available at hand, the finite frequency phase conjugation function yields an initial guess of the spatial support of the current source, which is subsequently optimized using l_1-regularization.

2. Formulation of the problem and preliminaries

Let $\Omega \subset \mathbb{R}^3$ be an open bounded domain with a Lipschitz boundary Γ. Consider the time-harmonic Maxwell's equations

$$\begin{cases} \nabla \times \mathbf{E}(\boldsymbol{x}, \omega) = \mathrm{i}\omega\mu_0\mathbf{H}(\boldsymbol{x}, \omega), & \boldsymbol{x} \in \mathbb{R}^3, \\ \nabla \times \mathbf{H}(\boldsymbol{x}, \omega) + \mathrm{i}\omega\epsilon_0\mathbf{E}(\boldsymbol{x}, \omega) = \mathbf{J}(\boldsymbol{x}), & \boldsymbol{x} \in \mathbb{R}^3, \end{cases} \tag{1}$$

subject to the Silver–Müller radiation conditions

$$\lim_{|\boldsymbol{x}|\to\infty} |\boldsymbol{x}|\big(\sqrt{\mu_0}\mathbf{H} \times \hat{\boldsymbol{x}} - \sqrt{\epsilon_0}\mathbf{E}\big) = \mathbf{0}, \quad \text{where } \hat{\boldsymbol{x}} := \boldsymbol{x}/|\boldsymbol{x}|, \tag{2}$$

with frequency pulsation ω, electric permittivity $\epsilon_0 > 0$ and magnetic permeability $\mu_0 > 0$, where $\mathbf{E}$ and $\mathbf{H}$ are the time-harmonic electric and magnetic fields, respectively. Here $\mathbf{J}(\boldsymbol{x}) \in \mathbb{R}^3$ is the current source density, assumed to be sufficiently smooth and compactly supported in Ω, that is, $\mathrm{supp}\{\mathbf{J}\} \subset\subset \Omega$.

Define the admissible set of frequencies and the boundary data

$$\mathcal{W} := (\omega_n)_{n=1}^N \quad \text{and} \quad \mathbf{d}(\boldsymbol{x}, \omega) = \mathbf{E}(\boldsymbol{x}, \omega) \quad \text{for all } (\boldsymbol{x}, \omega) \in \Gamma \times \mathbb{R}. \tag{3}$$

Then, the ultimate goal of this work is to tackle the following problem:

Given $\mathbf{d}_\mathcal{W} := \mathbf{d}|_{\Gamma\times\mathcal{W}}$ on broadband frequencies (i.e. for N sufficiently large), identify $\mathrm{supp}\{\mathbf{J}\}$ *of current source* $\mathbf{J}$.

We refer to $\kappa_0 := \omega\sqrt{\epsilon_0\mu_0}$ as the wave number and $c_0 := 1/\sqrt{\epsilon_0\mu_0}$ as the speed in dielectrics. We also let ν to be the outward unit normal to Γ. By (1),

$$\nabla \times \nabla \times \mathbf{E} - \kappa_0^2\mathbf{E} = \mathrm{i}\omega\mu_0\mathbf{J}(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathbb{R}^3. \tag{4}$$

Let $\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x}, \omega)$ be the outgoing electric–electric Green function for Maxwell's equations (1), that is,

$$\nabla \times \nabla \times \mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x}, \omega) - \kappa_0^2\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x}, \omega) = \mathrm{i}\omega\mu_0\mathbf{I}\delta_{\mathbf{0}}(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathbb{R}^3, \tag{5}$$

where $\delta_{\mathbf{0}}(\boldsymbol{x})$ is the Dirac mass at $\boldsymbol{x} = \mathbf{0}$. The following identities are the key ingredients to elucidate the localization and resolution limits of the imaging functional proposed in the next section. The variants of the identity in Lemma 2.1 can be found in [8,9] and [6].

Lemma 2.1 *(Electromagnetic Helmholtz–Kirchhoff identity). Let $\mathbf{B}(\mathbf{0}, R)$ be an open ball in $\mathbb{R}^3$ with radius R, center $\mathbf{0}$ and boundary $\partial\mathbf{B}(\mathbf{0}, R)$. Then, for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^3$,*

$$\lim_{R\to+\infty} \int_{\partial\mathbf{B}(0,R)} \mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x} - \xi, \omega)\overline{\mathbf{G}_0^{\mathrm{ee}}(\xi - \boldsymbol{y}, \omega)}\, \mathrm{d}\sigma(\xi) = \mu_0 c_0 \Re\mathrm{e}\big\{\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x} - \boldsymbol{y}, \omega)\big\},$$

where the superposed bar indicates a complex conjugate.

Lemma 2.2. *For all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^3$, $\boldsymbol{x} \neq \boldsymbol{y}$,*

$$\frac{\epsilon_0}{2\pi} \int_{\mathbb{R}} \Re\mathrm{e}\big\{\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x} - \boldsymbol{y}, \omega)\big\}\, \mathrm{d}\omega = \delta_{\boldsymbol{x}}(\boldsymbol{y})\mathbf{I}.$$

Proof. Let $\widehat{\mathbf{G}}$ be the solution to

$$\frac{1}{c_0^2}\frac{\partial^2 \widehat{\mathbf{G}}}{\partial t^2}(\boldsymbol{x},t;\boldsymbol{y},\tau)+\nabla\times\nabla\times\widehat{\mathbf{G}}(\boldsymbol{x},t;\boldsymbol{y},\tau)=-\delta_{\boldsymbol{y}}(\boldsymbol{x})\delta_\tau(t)\mathbf{I},\quad \boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^3,\ t>\tau, \tag{6}$$

such that for all constant vectors $\mathbf{p}\in\mathbb{R}^3$

$$\widehat{\mathbf{G}}(\boldsymbol{x},t;\boldsymbol{y},\tau)\mathbf{p}=\mathbf{0}=\frac{\partial\widehat{\mathbf{G}}}{\partial t}(\boldsymbol{x},t;\boldsymbol{y},\tau)\mathbf{p}\quad\text{for all } \boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^3,\ t<\tau$$

and let $\mathbf{G}(\boldsymbol{x},\boldsymbol{y},\omega)\mathbf{p}$ be the Fourier transform of $\widehat{\mathbf{G}}(\boldsymbol{x},t;\boldsymbol{y},0)\mathbf{p}$. Then, integrating (6) over $[\tau^-,\tau^+]$, using the causality conditions and continuity of $\widehat{\mathbf{G}}\mathbf{p}$ away from $t=\tau$, we have:

$$\frac{\partial\widehat{\mathbf{G}}}{\partial t}(x,t;y,\tau)\mathbf{p}\Big|_{t=\tau^+}=-c_0^2\delta_{\boldsymbol{y}}(\boldsymbol{x})\mathbf{p},\quad \boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^3,\ \boldsymbol{x}\neq\boldsymbol{y}.$$

Consequently, by Parseval's identity

$$\int_{\mathbb{R}} \mathrm{i}\omega\mathbf{G}(\boldsymbol{x},\boldsymbol{y},\omega)\mathbf{p}\,\mathrm{d}\omega=2c_0^2\pi\,\delta_{\boldsymbol{y}}(\boldsymbol{x})\mathbf{p}\int_{\mathbb{R}}\delta_0(\omega)\,\mathrm{d}\omega=2c_0^2\pi\,\delta_{\boldsymbol{y}}(\boldsymbol{x})\mathbf{p}. \tag{7}$$

Finally, relation (7) leads to the conclusion together with $\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x}-\boldsymbol{y},\omega)\mathbf{p}=-\mathrm{i}\omega\mu_0\mathbf{G}(\boldsymbol{x},\boldsymbol{y},\omega)\mathbf{p}$ and by varying and choosing $\mathbf{p}$ as the canonical basis vectors in $\mathbb{R}^3$. □

3. Source localization with a finite set of frequencies

As a first step towards the ultimate goal, we find the spatial support of the current source, $\operatorname{supp}\{\mathbf{J}\}$, from data $\mathbf{d}(\boldsymbol{x},\omega)$ with $\omega\in\mathbb{R}$. For a fixed $\omega\in\mathbb{R}$, define the adjoint field $\mathbf{E}^*$ to be the solution to

$$\nabla\times\nabla\times\mathbf{E}^*(\boldsymbol{x},\omega)-\kappa_0^2\mathbf{E}^*(\boldsymbol{x},\omega)=\mathrm{i}\omega\mu_0\overline{\mathbf{d}(\boldsymbol{x},\omega)}\delta_\Gamma(\boldsymbol{x}),\quad(\boldsymbol{x},\omega)\in\mathbb{R}^3\times\mathbb{R}, \tag{8}$$

where δ_Γ is the Dirac mass at the boundary Γ. The *phase conjugation* functional is defined by

$$\mathcal{I}(\boldsymbol{x}):=\frac{\epsilon_0}{2\pi c_0\mu_0}\int_{\mathbb{R}}\mathbf{E}^*(\boldsymbol{x},\omega)\,\mathrm{d}\omega,\quad\forall\boldsymbol{x}\in\mathbb{R}^3. \tag{9}$$

Then, $\mathcal{I}(\boldsymbol{x})$ yields $\operatorname{supp}\{\mathbf{J}\}$ approximately. In fact, we have the following theorem.

Theorem 3.1. *For $\boldsymbol{x}\in\Omega$ sufficiently far from Γ compared to the wavelength,*

$$\mathcal{I}(\boldsymbol{x})\simeq\mathbf{J}(\boldsymbol{x}).$$

Proof. Since $\mathbf{J}$ is supported compactly inside Ω, for all $\boldsymbol{x}\in\Omega$ and $\boldsymbol{y}\in\Gamma$, we have:

$$\mathbf{E}^*(\boldsymbol{x},\omega)=\int_\Gamma\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{y}-\boldsymbol{x},\omega)\overline{\mathbf{d}(\boldsymbol{y},\omega)}\,\mathrm{d}\sigma(\boldsymbol{y})\quad\text{and}\quad\mathbf{d}(\boldsymbol{y},\omega)=\int_\Omega\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{y}-\boldsymbol{z},\omega)\mathbf{J}(\boldsymbol{z})\,\mathrm{d}\boldsymbol{z}|_{\boldsymbol{y}\in\Gamma}. \tag{10}$$

Therefore, by using (10) in (9), we arrive at

$$\mathcal{I}(\boldsymbol{x})=\frac{\epsilon_0}{2\pi c_0\mu_0}\int_{\mathbb{R}^d}\int_{\mathbb{R}}\left(\int_\Gamma\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x}-\boldsymbol{y},\omega)\overline{\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{y}-\boldsymbol{z},\omega)}\,\mathrm{d}\sigma(\boldsymbol{y})\right)\mathrm{d}\omega\mathbf{J}(\boldsymbol{z})\,\mathrm{d}\boldsymbol{z}.$$

Now, let us invoke the Helmholtz–Kirchhoff identity from Lemma 2.1 and then Lemma 2.2 to get:

$$\begin{aligned}\mathcal{I}(\boldsymbol{x})&\simeq\int_{\mathbb{R}^d}\left(\frac{\epsilon_0}{2\pi}\int_{\mathbb{R}}\Re\mathrm{e}\big\{\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x}-\boldsymbol{z},\omega)\big\}\mathrm{d}\omega\right)\mathbf{J}(\boldsymbol{z})\,\mathrm{d}\boldsymbol{z}\\&\simeq\int_{\mathbb{R}^d}\delta_{\boldsymbol{x}}(\boldsymbol{z})\mathbf{J}(z)\,\mathrm{d}\boldsymbol{z}=\mathbf{J}(\boldsymbol{x}).\qquad\square\end{aligned}$$

Now, we address the inverse current source problem using boundary data $\mathbf{d}_{\mathcal{W}}=\mathbf{d}(\boldsymbol{x},\omega)|_{\Gamma\times\mathcal{W}}$. Inspired by the functional $\mathcal{I}$ defined in (9), we define a single frequency functional $\mathcal{I}_n$ by (12). However, since we are dealing with a finite set of

Algorithm 1 Fast iterative-shrinkage thresholding with backtracking.

Require: Set $\gamma_0 > 0$, $\eta > 1$, $\boldsymbol{x}_0 = \mathbf{0}$, $\boldsymbol{y}_1 = \boldsymbol{x}_0$, $s_1 = 1$.
for $m \geq 1$ **do**
  Set $i_m = 1$, $\beta = \eta\gamma_{m-1}$.
  while $\mathcal{L}(\mathcal{T}_\beta(\boldsymbol{y}_m), \lambda) > \mathcal{P}_\beta(\mathcal{T}_\beta(\boldsymbol{y}_m), \boldsymbol{y}_m)$ **do**
    Update $i_m = i_m + 1$, $\beta = \eta^{i_m}\gamma_{m-1}$.
  end while
  Set $\gamma_m = \beta$, $x_m = \mathcal{T}_{\gamma_m}(\boldsymbol{y}_m)$.
  Update $s_{m+1} = \frac{1}{2}(1 + \sqrt{1 + 4s_m^2})$, $\boldsymbol{y}_{m+1} = \boldsymbol{x}_m + (\frac{s_{m-1}}{s_{m+1}})(\boldsymbol{x}_m - \boldsymbol{x}_{m-1})$, $i_m = 0$, $m = m + 1$.
end for
return $\widehat{\mathbf{J}} = \boldsymbol{x}_m$.

frequency measurements, the lack of information over the entire spectrum induces noise and blurring in the reconstruction. In order to fix the problem, an initial guess to the current source density is identified in the sequel, which is then optimized providing an improved approximation to supp{**J**}.

Let $0 \leq \kappa_0^1 \leq \kappa_0^2 \leq \cdots \leq \kappa_0^N$ be N wave numbers corresponding to $\omega_n \in \mathcal{W}$ for $n = 1, 2, \cdots, N$. Let us define the adjoint field $\mathbf{E}_n^*$ corresponding to a fixed frequency $\omega_n \in \mathcal{W}$ to be the solution to

$$\nabla \times \nabla \times \mathbf{E}_n^*(\boldsymbol{x}, \omega_n) - \left(\kappa_0^n\right)^2 \mathbf{E}_n^*(\boldsymbol{x}, \omega_n) = \mathrm{i}\omega_n\mu_0 \overline{\mathbf{d}_{\mathcal{W}}(\boldsymbol{x}, \omega_n)}\delta_\Gamma(\boldsymbol{x}), \quad (\boldsymbol{x}, \omega_n) \in \mathbb{R}^3 \times \mathbb{R}, \tag{11}$$

and the *single frequency phase conjugation* functional by

$$\mathcal{I}_n(\boldsymbol{x}) := \frac{\epsilon_0}{2\pi c_0 \mu_0} \mathbf{E}_n^*(\boldsymbol{x}, \omega_n). \tag{12}$$

The following lemma holds.

Lemma 3.2. *For all $\boldsymbol{x} \in \Omega$ sufficiently far from Γ, we have*

$$\mathcal{I}_n(\boldsymbol{x}) \simeq \frac{\epsilon_0}{2\pi} \int_\Omega \Re e\left\{\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x} - \boldsymbol{y}, \omega_n)\right\}\mathbf{J}(\boldsymbol{y})\,\mathrm{d}\boldsymbol{y}.$$

Proof. The proof is similar to that of Theorem 3.1. Indeed, we conclude from Lemma 2.1 and relation

$$\begin{aligned}\mathcal{I}_n(\boldsymbol{x}) &= \frac{\epsilon_0}{2\pi c_0 \mu_0} \mathbf{E}_n^*(\boldsymbol{x}, \omega_n) \\ &= \frac{\epsilon_0}{2\pi c_0 \mu_0} \int_\Omega \int_\Gamma \mathbf{G}_0^{\mathrm{ee}}(\xi - \boldsymbol{x}, \omega_n) \overline{\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{y} - \xi, \omega_n)}\,\mathrm{d}\sigma(\xi)\mathbf{J}(\boldsymbol{y})\,\mathrm{d}\boldsymbol{y}. \qquad \square\end{aligned}$$

Now, the aim is to utilize an l_1-regularization to optimize the localization of the current source density. The objective is to resolve the following optimization problem:

$$\mathbf{J}_\lambda(\boldsymbol{x}) := \underset{\widehat{\mathbf{J}} \in \mathbb{R}^d}{\arg\min}\, \mathcal{M}(\widehat{\mathbf{J}}) + \mathcal{R}(\widehat{\mathbf{J}}), \tag{13}$$

where

$$\mathcal{M}(\widehat{\mathbf{J}}) := \frac{1}{2N} \sum_{n=1}^N \left\| \mathcal{I}_n(\boldsymbol{x}) - \frac{\epsilon_0}{2\pi} \int_\Omega \Re e\left\{\mathbf{G}_0^{\mathrm{ee}}(\boldsymbol{x} - \boldsymbol{y}, \omega_n)\right\}\widehat{\mathbf{J}}(\boldsymbol{y})\,\mathrm{d}\boldsymbol{y} \right\|^2, \tag{14}$$

$$\mathcal{R}(\widehat{\mathbf{J}}) := \lambda \left\|\widehat{\mathbf{J}}(\boldsymbol{x})\right\|_{l_1}. \tag{15}$$

Here the first term $\mathcal{M}$ is the data fidelity term and the second term $\mathcal{R}$ accounts for the l_1-regularization. It is clarified that λ is a regularization parameter controlling the relative weights of the two terms and provides a trade-off between fidelity to the measurements and noise sensitivity. Here $\|\cdot\|$ denotes the Euclidean norm in $\mathbb{R}^3$.

The direct computations of the solution $\mathbf{J}_\lambda$ to the minimization problem (13) are not trivial. Thus, in order to obtain $\mathbf{J}_\lambda$ explicitly, approximation schemes are indispensable. For this, we follow Beck and Teboulle [7] and use their *fast iterative shrinkage thresholding algorithm with backtracking*; see Algorithm 1. This method belongs to the class of split gradient descent iterative schemes with a global convergence rate $O(m^{-2})$, where m is the iteration counter.

For any $\gamma > 0$, define the quadratic approximation of the Lagrangian $\mathcal{L}(\widehat{\mathbf{J}}, \lambda) = \mathcal{M}(\widehat{\mathbf{J}}) + \mathcal{R}(\widehat{\mathbf{J}})$ by

$$\mathcal{P}_\gamma(\boldsymbol{x}, \boldsymbol{y}) := \mathcal{M}(\boldsymbol{y}) + \left\langle \boldsymbol{x} - \boldsymbol{y}, \nabla\mathcal{M}(\boldsymbol{y})\right\rangle + \frac{\gamma}{2}\|\boldsymbol{x} - \boldsymbol{y}\|^2 + \mathcal{R}(\boldsymbol{x}) \quad \text{and} \quad \mathcal{T}_\gamma(\boldsymbol{y}) := \underset{\boldsymbol{x} \in \mathbb{R}^d}{\arg\min}\left\{\mathcal{P}_\gamma(\boldsymbol{x}, \boldsymbol{y})\right\}, \tag{16}$$

where $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^3$ and $\langle\cdot,\cdot\rangle$ is the usual Euclidean inner product in $\mathbb{R}^3$.

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