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Disjoint mixing composition operators on the Hardy space in the unit ball ☆

Opérateurs de composition disjointement mélangeants sur l'espace de Hardy de la boule unité

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ABSTRACT

We characterize disjoint mixing and disjoint hypercyclicity of finite many composition operators acting on the Hardy space on the unit ball.

RÉSUMÉ

Nous caractérisons les propriétés de mélange disjoint et d'hypercyclicité disjointe d'une famille finie d'opérateurs de composition agissant sur l'espace de Hardy de la boule unité.

1. Introduction

Throughout this paper, let $n \geqslant 1$ be a fixed integer and $\mathbb{N}$ the set of all nonnegative integers. Let $\mathbb{B}$ be the unit ball of the complex n-dimensional Euclidean space $\mathbb{C}^n$ and $\partial\mathbb{B}$ be the boundary of the unit ball. Let $H(\mathbb{B})$ denote the collection of all holomorphic functions defined on $\mathbb{B}$, $LFT(\mathbb{B})$ the collection of all linear fractional maps of $\mathbb{B}$ and $Aut(\mathbb{B})$ denote the automorphic group of $\mathbb{B}$.

For $a \in \mathbb{B}$, $\varphi_a \in Aut(\mathbb{B})$ is the Möbius transformation defined by

$$\varphi_a(z) = \frac{a - P_a(z) - s_a Q_a(z)}{1 - \langle z, a\rangle}, \quad z \in \mathbb{B},$$

where $s_a = \sqrt{1-|a|^2}$, P_a is the orthogonal projection from $\mathbb{C}^n$ onto the one-dimensional subspace $[a]$ generated by a, and $Q_a = I - P_a$ is the projection onto the orthogonal complement of $[a]$, that is,

$$P_a(z) = \frac{\langle z, a\rangle}{|a|^2}a, \qquad Q_a(z) = z - P_a(z), \quad z \in \mathbb{B}.$$

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When $a = 0$, we simply define $\varphi_a(z) = -z$.

The classical Hardy space $H^p = H^p(\mathbb{B})$ for $0 < p < \infty$, consists of all $f \in H(\mathbb{B})$, satisfying the norm condition

$$\|f\|_{H^p}^p = \sup_{0<r<1} \int_{\partial\mathbb{B}} \left|f(r\zeta)\right|^p d\sigma(\zeta) < \infty,$$

where $d\sigma$ is the normalized Lebesgue measure on the boundary $\partial\mathbb{B}$. This space is the most well-known and widely studied space of holomorphic functions. When $1 \leqslant p < \infty$, H^p is a Banach space under the norm $\|.\|_{H^p}$.

As we all know $H^2(\mathbb{B})$ is a Hilbert space with the inner product

$$\langle f, g\rangle = \int_{\partial\mathbb{B}} f(\zeta)\overline{g(\zeta)}\, d\sigma(\zeta).$$

And its norm is defined by

$$\|f\|^2 = \int_{\partial\mathbb{B}} \left|f(\zeta)\right|^2 d\sigma(\zeta) < \infty. \tag{1}$$

The composition operator induced by an analytic self-map φ of the unit ball $\mathbb{B}$ is defined as follows,

$$C_\varphi f = f \circ \varphi, \quad f \in H(\mathbb{B}).$$

This operator was well studied for many years, readers interested in this topic can refer to the books [22] by Shapiro, [13] by Cowen and MacCluer and the paper [1], which are excellent sources for the development of the theory of composition operators and function spaces.

Let $L(X)$ denote the space of all linear and continuous operators on a separable, infinite dimensional Banach space X. A continuous linear operator $T \in L(X)$ is said to be *hypercyclic* if there is an $f \in X$ such that the orbit

$$Orb(T, f) = \left\{T^n f\colon n = 0, 1, \ldots\right\}$$

is dense in X. Such a vector f is said to be *hypercyclic* for T. It is well known that an operator T on a separable Banach space X is *hypercyclic* if and only if it is *topologically transitive* in the sense of dynamical systems, i.e. for every pair of non-empty open subsets U and V of X there is $n \in \mathbb{N}$ that $T^n(U) \cap V \neq \emptyset$. There is an important criterion to show whether T is hypercyclic or not. We refer the readers to the paper [3]. A stronger condition is the following: the operator T on X is called *topologically mixing* if, for every pair of non-empty open subsets U and V of X, there is $N \in \mathbb{N}$ such that $T^n(U) \cap V \neq \emptyset$ for each $n \geqslant N$.

For motivation, examples and background about linear dynamics we refer the reader to the books [2] by Bayart and Matheron, [16] by Grosse-Erdmann and Manguillot, and the article by Godefroy and Shapiro [15].

For a positive integer n and $N \geqslant 2$, the n-th iterate of φ_i, denoted by $\varphi_i^{[n]}$ for $i = 1, \ldots, N$, is the function obtained by composing φ_i with itself n times; also, φ_0 is defined as the identity function. Besides, if φ_i is invertible, we can define the iterates $\varphi_i^{[-n]} = \underbrace{\varphi_i^{-1} \circ \varphi_i^{-1} \circ \cdots \circ \varphi_i^{-1}}_{n \text{ times}}$ for $i = 1, \ldots N$.

2. Some definitions

In 2007, Bès and Peris and, independently, Beral investigate the property of the orbits

$$\left\{(z, z, \ldots, z), (T_1 z, T_2 z, \ldots, T_N z), \left(T_1^2 z, T_2^2 z, \ldots, T_N^2 z\right), \ldots\right\} \quad (z \in X)$$

for $N \geqslant 2$. We refer the interested readers to the recent papers [4] and [7]. They study the case when one of these orbits is dense in X^N endowed with the product topology for some $z \in X$. If there is some vector satisfying the above condition, the operators $T_1, \ldots, T_N$ are called *disjoint hypercyclic*, which is a weaker notion than the notion of disjointness of Furstenberg (see, e.g. [14]). Next we list some definitions.

Definition 1. (See [20, Definition 1.3.1].) For $N \geqslant 2$, we say that N sequences of operators $(T_{1,n})_{n=1}^\infty, \ldots, (T_{N,n})_{n=1}^\infty$ in $L(X)$ are disjoint hypercyclic or d-hypercyclic provided that the sequence of direct sums $(T_{1,n} \oplus \cdots \oplus T_{N,n})_{n=1}^\infty$ has a hypercyclic vector of the form $(x, \ldots, x) \in X^N$. Then x is called a d-hypercyclic vector for the sequences $(T_{1,n})_{n=1}^\infty, \ldots, (T_{N,n})_{n=1}^\infty$. The operators $T_1, \ldots, T_N$ in $L(X)$ are called disjoint hypercyclic if the sequences of iterations $(T_1^n)_{n=1}^\infty, \ldots, (T_N^n)_{n=1}^\infty$ are disjoint hypercyclic.

It is well known that two *d-hypercyclic* operators must be substantially different (see, e.g., [7]). For example, an operator cannot be *d-hypercyclic* with a scalar multiple of itself. Recent results on *disjointness in hypercyclicity* include the works by Salas [21], Shkarin [23], Bès et al. [5,8], and so on.

Definition 2. (See [7, Definition 2.1].) For $N \geqslant 2$, we say that N given operators $T_1, \dots, T_N$ on a separable, infinite dimensional Banach space X are d-mixing provided that, for every non-empty open subsets $V_0, \dots, V_N$ of X, there exists $m \in \mathbb{N}$ such that

$$V_0 \cap T_1^{-j}(V_1) \cap \cdots \cap T_N^{-j}(V_N) \neq \emptyset$$

for each $j \geqslant m$.

When the space X is Baire, a simple Baire Category argument shows that $T_1, \dots, T_N$ are *d-hypercyclic* whenever they are *d-mixing*.

Definition 3. (See [7, Definition 2.5].) Let (n_k) be an increasing sequence of positive integers. We say that $N \geqslant 2$ operators $T_1, \dots, T_N$ in $L(X)$ satisfy the d-Hypercyclicity Criterion with respect to (n_k) provided that there exist dense subsets $X_0, \dots, X_N$ of X and mappings $S_{l,k} : X_l \to X$ ($k \in \mathbb{N}$, $1 \leqslant l \leqslant N$) satisfying

(i) $T_l^{n_k} \underset{k \to \infty}{\to} 0$ pointwise on X_0,

(ii) $S_{l,k} \underset{k \to \infty}{\to} 0$ pointwise on X_l, and

(iii) $\left(T_l^{n_k} S_{i,k} - \delta_{i,l} Id_{X_l}\right) \underset{k \to \infty}{\to} 0$ pointwise on $X_l \quad (1 \leqslant i \leqslant N)$.

In general, we say that $T_1, \dots, T_N$ satisfy the d-Hypercyclicity Criterion if there exists some increasing sequence of positive integers (n_k) for which the above conditions are satisfied.

Proposition 1. *(See [7, Proposition 2.6].) Let $T_1, \dots, T_N$ satisfy the d-Hypercyclicity Criterion with respect to a sequence (n_k), where $N \geqslant 2$. Then the sequences $\{T_1^{n_k}\}_{k=1}^{\infty}, \dots, \{T_N^{n_k}\}_{k=1}^{\infty}$ are d-mixing. In particular, $T_1, \dots, T_N$ are d-hypercyclic. Indeed, if $(n_k) = (k)$, then $T_1, \dots, T_N$ are d-mixing.*

In [11], Chen et al. proved that the composition operator C_φ is *hypercyclic* on $H^2(\mathbb{B})$ if φ is an automorphism of $\mathbb{B}$ without interior fixed point. If an analytic map $\varphi : \mathbb{B} \to \mathbb{B}$ has more than two fixed points in $\overline{\mathbb{B}}$, we know that C_φ is *non-cyclic* and it is *hypercyclic* if and only if its differential is injective at some point when φ is not an automorphism and has exactly two boundary fixed points by [9]. Moreover, let us observe that if an analytic map $\varphi : \mathbb{B} \to \mathbb{B}$ has a unique boundary fixed point with the boundary dilation coefficient 1, and if the restriction of φ to any non-trivial affine subset of $\mathbb{B}$ is not an automorphism, then C_φ is not *hypercyclic*.

For a single composition operator, Bourdon and Shapiro [10] completely characterized the *cyclic* and *hypercyclic* composition operators on $H^2(\mathbb{D})$ induced by the linear fractional maps, in accordance with fixed-points location. The recent paper [17] gave a characterization of the *cyclic* behavior of the linear fractional composition operators in the unit ball of $\mathbb{C}^N$. As regard to the *d-hypercyclicity* of N composition operators, the following Theorem A gives a sufficient condition for the disjointness of N hypercyclic composition operators on $H^2(\mathbb{D})$ (see, e.g., [20]).

Theorem A. *Let $C_{\varphi_1}, \dots, C_{\varphi_N}$ be* hypercyclic *composition operators on $H^2(\mathbb{D})$, where $\varphi_1, \dots, \varphi_N \in LFT(\mathbb{D})$ and $N \geqslant 2$. Suppose that for each $1 \leqslant l, j \leqslant N$ with $l \neq j$, we have*

$$\left(\varphi_l^{[-n]} \circ \varphi_j^{[n]}\right)(z) \to \gamma_l, \quad n \to \infty,$$

for almost all $z \in \partial\mathbb{D}$, where γ_l is a fixed point of φ_l. Then $C_{\varphi_1}, \dots, C_{\varphi_N}$ are d-hypercyclic. *Moreover, $C_{\varphi_1}, \dots, C_{\varphi_N}$ satisfy the* d-Hypercyclicity Criterion *with respect the sequence $(n_k) = (k)$, and are thus* d-mixing.

In this paper, we will generalize the above result to the unit ball of $\mathbb{C}^n$ under some conditions. The proofs of the present paper are partially based on Martin's work in [20]. But such a characterization would be difficult for the high dimension cases, some properties are not easily managed. We need some new methods and calculating techniques.

3. Some lemmas

In this section, we cite some lemmas, which will be used in the proof of the main theorems.

Lemma 1. *(See [11, Theorem 3].) Suppose $\varphi \in Aut(\mathbb{B})$. Then the composition operator C_φ is hypercyclic on $H^2(\mathbb{B})$ if and only if φ has no fixed points in $\mathbb{B}$.*

Lemma 2. *(See [13, Theorem 2.83].) Suppose $\varphi : \mathbb{B} \to \mathbb{B}$ is an analytic map with no fixed points in $\mathbb{B}$, then there is a point $\zeta \in \partial\mathbb{B}$, such that the iterate*

$$\varphi^{[k]} = \underbrace{\varphi \circ \cdots \circ \varphi}_{k\ times}$$

converges uniformly to ζ on any compact subset of $\mathbb{B}$.

The boundary point ζ will be called the *Denjoy–Wolff point* of φ. In addition, the following inequality can be obtained from [19, Theorem 1.3]

$$0 < \liminf_{z \to \zeta} \frac{1 - |\varphi(z)|^2}{1 - |z|^2} = \delta \leqslant 1.$$

The real number δ is referred to as the *boundary dilation coefficient* of φ.

Remark. An analytic self-map φ of $\mathbb{B}$ will be called *elliptic* if φ fixes some point of $\mathbb{B}$, *parabolic* if φ has no interior fixed point and boundary dilation coefficient $\delta = 1$, and *hyperbolic* if φ has no interior fixed point and boundary dilation coefficient $\delta < 1$.

Lemma 3. *(See [12, Lemma 4.3].) Let X be the set of polynomials vanishing at the boundary point w. Then X is a dense subset of $H^2(\mathbb{B})$.*

We show the following lemma, a modification of Lemma 3.

Lemma 4. *Let m be a finite positive integer. Define the finite set*

$$A = \{\zeta_1, \ldots, \zeta_m \colon \zeta_i \in \partial\mathbb{B},\ i = 1, \ldots, m\}.$$

Then the set of polynomials vanishing on A is dense in $H^2(\mathbb{B})$.

Proof. Let Y be the set of polynomials vanishing on A. We will show $\overline{Y} = H^2(\mathbb{B})$. Let X_0 denote the set of polynomials in $H^2(\mathbb{B})$. It is well known that $\overline{X_0} = H^2(\mathbb{B})$. Then from the proof of the above Lemma 3, it follows that the set $(\langle z, e_1\rangle - 1)X_0$ is dense in $H^2(\mathbb{B})$, where $e_1 = \{1, 0, \ldots, 0\}$, $z \in \mathbb{B}$. For a general $\zeta_1 \in \partial\mathbb{B}$, there is a unitary transformation U such that $Ue_1 = \zeta_1$. Then the set

$$\big(\langle z, e_1\rangle - 1\big)X_0 = \big(\langle Uz, Ue_1\rangle - 1\big)X_0 = \big(\langle Uz, \zeta_1\rangle - 1\big)X_0 = \big(\langle \tilde{z}, \zeta_1\rangle - 1\big)X_0$$

is dense in $H^2(\mathbb{B})$, since the polynomial $P \circ U \in X_0$ for every polynomial $P \in X_0$. We should note that $\langle \tilde{z}, \zeta_1\rangle - 1 = 0$ vanishes at $\tilde{z} = \zeta_1 \in \partial\mathbb{B}$.

Inductively, assume the lemma holds for the sets $m-1 \geqslant 1$ elements $\{\zeta_1, \ldots, \zeta_{m-1}\}$. Then for the set $A = \{\zeta_1, \ldots, \zeta_m \colon \zeta_i \in \partial\mathbb{B},\ i = 1, \ldots, m\}$.

Since the function $\langle z, \zeta_m\rangle - 1$ vanishing at $\zeta_m \in \partial\mathbb{B}$, we know that $(\langle z, \zeta_m\rangle - 1)X_0$ is dense in $H^2(\mathbb{B})$ from above. Thus it is sufficient to verify that Y is dense in $(\langle z, \zeta_m\rangle - 1)X_0$.

Suppose g is an arbitrary element in $(\langle z, \zeta_m\rangle - 1)X_0$, then $g(z) = (\langle z, \zeta_m\rangle - 1)f \in (\langle z, \zeta_m\rangle - 1)X_0$, where $f \in X_0 \subset H^2(\mathbb{B})$. By inductive hypothesis, there exists a sequence (p_k) of polynomials vanishing on $\{\zeta_1, \ldots, \zeta_{m-1},\ \zeta_i \in \partial\mathbb{B},\ i = 1, \ldots, m-1\}$ so that $p_k \to f$ on $H^2(\mathbb{B})$. It is obvious that the polynomials $q_k(z) = (\langle z, \zeta_m\rangle - 1)p_k(z)$, $k \in \mathbb{N}$ belong to Y, and $q_k \to g$ on $H^2(\mathbb{B})$. Then Y is dense in $H^2(\mathbb{B})$. This completes the proof. □

In the present paper, we generalize Theorem A to the unit ball $\mathbb{B}$ and we get the sufficient conditions for the *d-hypercyclicity* and *d-mixing* of N composition operators $C_{\varphi_1}, \ldots, C_{\varphi_N}$ on the Hardy space $H^2(\mathbb{B})$, where $\varphi_1, \ldots, \varphi_N \in Aut(\mathbb{B})$ for $N \geqslant 2$.

4. Main theorem

Theorem 1. *Let $C_{\varphi_1}, \ldots, C_{\varphi_N}$ be hypercyclic composition operators on $H^2(\mathbb{B})$, where $\varphi_1, \ldots, \varphi_N \in Aut(\mathbb{B})$ and $N \geqslant 2$. Suppose that for each $1 \leqslant l, j \leqslant N$ with $l \neq j$, we have that*

$$\big(\varphi_l^{[-n]} \circ \varphi_j^{[n]}\big)(z) \to \gamma_l, \quad n \to \infty, \tag{2}$$

for almost all $z \in \partial\mathbb{B}$, where γ_l is a fixed point of φ_l. Then the operators $C_{\varphi_1}, \ldots, C_{\varphi_N}$ satisfy the d-Hypercyclicity Criterion with respect to $(n_k) = (k)$, thus are d-mixing. In particular, they are d-hypercyclic.

Proof. For every $1 \leqslant l \leqslant N$ and $\varphi_l \in Aut(\mathbb{B})$, we have that $\varphi_l^{-1} \in Aut(\mathbb{B})$. Since C_{φ_l} is a hypercyclic composition operator on $H^2(\mathbb{B})$, then φ_l fixes no interior point in $\mathbb{B}$ from Lemma 1. Further using Lemma 2, there are $\zeta_l, \eta_l \in \partial\mathbb{B}$ such that

$$\varphi_l^{[n]}(z) \to \zeta_l, \qquad \varphi_l^{[-n]}(z) \to \eta_l, \quad n \to \infty, \ z \in \mathbb{B},$$

where ζ_l and η_l are the attractive and the repellent fixed points of φ_l, respectively, if φ_l is hyperbolic and $\zeta_l = \eta_l$ if φ_l is parabolic $(1 \leqslant l \leqslant N)$.

Denote

$$A = \{\zeta_1, \ldots, \zeta_N\}.$$

By Lemma 4, the set X_0 of polynomials that vanishing on A is dense in $H^2(\mathbb{B})$. Since $\varphi_l^{[n]} \to \zeta_l$, $f \circ \varphi_l^{[n]} \to f(\zeta_l) = 0$ for every $f \in X_0$. Besides $\|f \circ \varphi_l^{[n]}\|_\infty \leqslant \|f\|_\infty$, further by the integral representation of the $H^2(\mathbb{B})$-norm, for every $f \in X_0$ it follows that

$$\|C_{\varphi_l}^n f\|^2 = \int_{\partial\mathbb{B}} \left|f \circ \varphi_l^{[n]}(\xi)\right|^2 \mathrm{d}\sigma(\xi) \to 0, \quad n \to \infty.$$

That is

$$C_{\varphi_l}^n \underset{n\to\infty}{\to} 0 \text{ pointwise on } X_0 \quad (1 \leqslant l \leqslant N). \tag{3}$$

Next, for each $1 \leqslant l \leqslant N$, let X_l denote the set of polynomials that are vanishing on $\{\zeta_l, \eta_l\}$. Let

$$S_{l,n} = C_{\varphi_l}^{-n} = C_{\varphi_l^{[-n]}} \quad (n \in \mathbb{N}).$$

By Lemma 4, X_l is dense in $H^2(\mathbb{B})$.

Similarly, since $f \circ \varphi_l^{[-n]} \to f(\eta_l) = 0$, $n \to \infty$, $f \in X_l$. Then we have that

$$\|S_{l,n} f\|^2 = \|C_{\varphi_l^{[-n]}} f\|^2 = \int_{\partial\mathbb{B}} \left|f \circ \varphi_l^{[-n]}(\xi)\right|^2 \mathrm{d}\sigma(\xi) \to 0, \quad n \to \infty.$$

That is,

$$S_{l,n} \underset{n\to\infty}{\to} 0 \text{ pointwise on } X_l \quad (1 \leqslant l \leqslant N). \tag{4}$$

It remains to show that

(iii) $C_{\varphi_j}^n S_{l,n} - \delta_{j,l} Id_{X_l} \underset{n\to\infty}{\to} 0$ pointwise on $X_l \quad (1 \leqslant j \leqslant N)$.

When $j = l$,

$$C_{\varphi_l}^n S_{l,n} = Id_{X_l} \text{ pointwise on } X_l. \tag{5}$$

When $j \neq l$, using (2) we have that

$$\|C_{\varphi_j}^n C_{\varphi_l}^{-n} f\|^2 = \int_{\partial\mathbb{B}} \left|f \circ \varphi_l^{[-n]} \circ \varphi_j^{[n]}(\xi)\right|^2 \mathrm{d}\sigma(\xi) \underset{n\to\infty}{\to} 0 \quad (1 \leqslant j \leqslant N), \tag{6}$$

pointwise on X_l.

From (3)–(6), it follows that the operators $C_{\varphi_1}, \ldots, C_{\varphi_N}$ satisfy the *d-Hypercyclicity Criterion* with respect to all the positive integer (n), thus are d-mixing. In particular, they are d-hypercyclic. This completes the proof. □

As an immediate consequence of Theorem 1 we have Theorem 2.

Theorem 2. *Let $C_{\varphi_1}, \ldots, C_{\varphi_N}$ be hypercyclic composition operators on $H^2(\mathbb{B})$, where $\varphi_1, \ldots, \varphi_N \in Aut(\mathbb{B})$ and $N \geqslant 2$. If the attractive fixed points of $\varphi_1, \ldots, \varphi_N$ are all distinct, then $C_{\varphi_1}, \ldots, C_{\varphi_N}$ are d-mixing, and thus d-hypercyclic.*

Proof. For $l \neq j$,

$$\varphi_l^{[n]} \to \zeta_l \text{ uniformly on compact subsets of } \partial\mathbb{B} \setminus \{\eta_l\}, \tag{7}$$

and

$$\varphi_j^{[n]} \to \zeta_j \text{ uniformly on compact subsets of } \partial\mathbb{B} \setminus \{\eta_j\}, \tag{8}$$

where ζ_l and η_l are the attractive and the repellent fixed points of φ_l, respectively, if φ_l is hyperbolic and $\zeta_l = \eta_l$ if φ_l is parabolic ($1 \leqslant l \leqslant N$).

From the assumption, since $\zeta_l \neq \zeta_j$, it follows that

$$\varphi_l^{[-n]} \circ \varphi_j^{[n]} \to \eta_l, \quad n \to \infty, \tag{9}$$

for almost $z \in \partial\mathbb{B}$. Then $C_{\varphi_1}, \dots, C_{\varphi_N}$ satisfy the hypothesis of Theorem 1. The desired result follows. □

5. Scalar multiples of composition operators on $H(\mathbb{B})$

Next we give a sufficient condition for the *d-mixing* of N scalar multiples of composition operators.

Lemma 5. *(See [5, Theorem 2.1].) Let $(\varphi_{1,n})_{n=1}^{\infty}, \dots, (\varphi_{N,n})_{n=1}^{\infty}$ be $N \geqslant 2$ sequences of holomorphic self-maps of a simply connected domain Ω. Then $(C_{\varphi_{1,n}})_{n=1}^{\infty}, \dots, (C_{\varphi_{N,n}})_{n=1}^{\infty}$ are d-hypercyclic on $H(\Omega)$ if and only if their sequences of symbols are injectively d-run-away, that is, they satisfy that for each compact $K \subset \Omega$, there exists $n \geqslant 1$ such that*

(i) *the sets $K, \varphi_{1,n}(K), \dots, \varphi_{N,n}(K)$ are pairwise disjoint, and*
(ii) *each of $\varphi_{1,n}, \dots, \varphi_{N,n}$ is injective on K.*

Then next lemma is the *Oka–Weil Theorem* which is an extension of *Runge's Theorem* to functions of several complex variables. The notation

$$\hat{K} := \{z \in \mathbb{C}^n: |p(z)| \leqslant \|p\|_K \text{ for all holomorphic polynomials } p\}$$

is the polynomial hull of K, where $\|p\|_K = \sup_{z\in K} |p(z)|$.

Lemma 6. *(See [18, Theorem OW].) Let $K \subset \mathbb{C}^n$ be compact with $K = \hat{K}$. Then for any function f holomorphic on a neighborhood of K, there exists a sequence $\{p_n\}$ of polynomials which converges uniformly to f on K.*

This result was first proved by André Weil in 1935 by using a multivariate generalization of the Cauchy integral formula for certain polynomial polyhedra. We refer the interested readers to Section 3 in [18].

Using Lemma 5, Lemma 6 and the similar proof of [6, Proposition 16], we have the following result.

Theorem 3. *Assume that $N \geqslant 2$ composition operators $C_{\varphi_1}, \dots, C_{\varphi_N}$ are d-mixing (respectively, d-hypercyclic) on $H(\mathbb{B})$. Then for any nonzero scalars $\mu_1, \dots, \mu_N$, the operators $\mu_1 C_{\varphi_1}, \dots, \mu_N C_{\varphi_N}$ are also d-mixing (respectively, d-hypercyclic) on $H(\mathbb{B})$.*

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