C. R. Acad. Sci. Paris, Ser. I 348 (2010) 439–444

Contents lists available at ScienceDirect

C. R. Acad. Sci. Paris, Ser. I

www.sciencedirect.com

Probability Theory

Reflected backward doubly stochastic differential equations driven by a Lévy process ☆

Équations différentielles doublement stochastiques rétrogrades réfléchies gouvernées par un processus de Lévy

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A R T I C L E   I N F O

Article history:
Received 24 September 2009
Accepted after revision 5 November 2009

Presented by Marc Yor

A B S T R A C T

We prove the existence and uniqueness of a solution for reflected backward doubly stochastic differential equations (RBDSDEs) driven by Teugels martingales associated with a Lévy process, in which the obstacle process is right continuous with left limits (càdlàg), via Snell envelope and the fixed point theorem.

R É S U M É

On démontre l'existence et l'unicité de la solution d'équations différentielles doublement stochastiques rétrogrades réfléchies (RBDSDE) gouvernées par des martingales de Teugels associées à un processus de Lévy dans lequel le processus obstacle est continu à droite et possède une limite à gauche (càdlàg), via l'enveloppe de Snell et un théorème de point fixe.

Version française abrégée

Les résultats les plus importants de cette Note sont les deux théorèmes suivants :

Théorème 1. *Si les fonctions* f *et* g *ne dépendent pas de* (Y, Z), *c'est-à-dire* $f(\omega, t, y, z) = f(\omega, t)$, $g(\omega, t, y, z) = g(\omega, t)$, *si* (H1) *est satisfaite, si les fontions* f, g *vérifient* $f \in \mathcal{H}^2$, $g \in \mathcal{H}^2$ *et si* (H4) *est satisfaite, alors il existe un triplet* $(Y_t, Z_t, K_t)_{0 \leqslant t \leqslant T}$ *solution de l'équation RBDSDE* (1) *correspondant aux données* (ξ, f, g, S).

Théorème 2. *On suppose les hypothèses* (H1)–(H4) *satisfaites, alors pour les données* (ξ, f, g, S) *l'équation RBDSDE* (1) *a une solution unique,* $(Y_t, Z_t, K_t)_{0 \leqslant t \leqslant T}$.

☆ The work is supported by the National Natural Science Foundation of China (Project 10901003) and the Great Research Project of Natural Science Foundation of Anhui Provincial Universities.

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1631-073X/$ – see front matter
doi:10.1016/j.crma.2009.11.004

1. Introduction

Very recently, Bahlali et al. [1] proved the existence and uniqueness of a solution to the following reflected backward doubly stochastic differential equations (RBDSDEs) with one continuous barrier and uniformly Lipschitz coefficients:

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)\,\mathrm{d}s + \int_t^T g(s, Y_s, Z_s)\,\mathrm{d}B_s + K_T - K_t - \int_t^T Z_s\,\mathrm{d}W_s^{(i)}, \quad 0 \leqslant t \leqslant T,$$

where the $\mathrm{d}W$ is a forward Itô integral and the $\mathrm{d}B$ is a backward Itô integral.

Motivated by [1–3,5–8], in this Note, we mainly consider the following RBDSDEs driven by Teugels martingales associated with a Lévy process, in which the obstacle process is right continuous with left limits (càdlàg):

$$Y_t = \xi + \int_t^T f(s, Y_{s-}, Z_s)\,\mathrm{d}s + \int_t^T g(s, Y_{s-}, Z_s)\,\mathrm{d}B_s + K_T - K_t - \sum_{i=1}^{\infty}\int_t^T Z_s^{(i)}\,\mathrm{d}H_s^{(i)}, \quad 0 \leqslant t \leqslant T, \tag{1}$$

where the $\mathrm{d}H^{(i)}$ is a forward semi-martingale Itô integrals [4] and the $\mathrm{d}B$ is a backward Itô integral.

The Note is devoted to prove the existence and uniqueness of a solution for RBDSDEs driven by a Lévy process. We hope to give the probabilistic interpretation of solutions for the obstacle problem for stochastic partial-differential equations in our further study by RBDSDEs proposed in this Note.

The Note is organized as follows. In Section 2, we give some preliminaries and notations. Section 3 is to prove the main results.

2. Preliminaries and notations

Let $(\Omega, \mathcal{F}, P, \mathcal{F}_t, B_t, L_t\colon t \in [0,T])$ be a complete Brownian–Lévy space in $\mathbb{R} \times \mathbb{R} \setminus \{0\}$, with Lévy measure ν, i.e. $(\Omega, \mathcal{F}, P)$ is a complete probability space, $\{B_t\colon t \in [0,T]\}$ is a standard Brownian motion in $\mathbb{R}$ and $\{L_t\colon t \in [0,T]\}$ is a $\mathbb{R}$-valued pure jump Lévy process of the form $L_t = bt + l_t$ independent of $\{B_t\colon t \in [0,T]\}$, which corresponds to a standard Lévy measure ν satisfying the following conditions:

(1) $\int_{\mathbb{R}} (1 \wedge y^2)\nu(\mathrm{d}y) < \infty$;
(2) $\int_{]-\varepsilon,\varepsilon[^c} e^{\lambda|y|}\nu(\mathrm{d}y) < \infty$, for every $\varepsilon > 0$ and for some $\lambda > 0$.

For each $t \in [0,T]$, we define the σ-field $\mathcal{F}_t$ by $\mathcal{F}_{0,t}^L$ and $\mathcal{F}_{t,T}^B$

$$\mathcal{F}_t \triangleq \mathcal{F}_{t,T}^B \vee \mathcal{F}_{0,t}^L,$$

where for any process $\{\eta_t\}$, $\mathcal{F}_{s,t}^{\eta} = \sigma\{\eta_r - \eta_s\colon s \leqslant r \leqslant t\} \vee \mathcal{N}$ and $\mathcal{N}$ is the class of P-null sets of $\mathcal{F}$. Note that $\{\mathcal{F}_t, t \in [0,T]\}$ is neither increasing nor decreasing, so it does not constitute a filtration.

Let us introduce some spaces:

- $\mathcal{H}^2 = \{(\varphi_t)_{0\leqslant t\leqslant T}\colon$ an $\mathcal{F}_t$-progressively measurable, real-valued process such that $E\int_0^T |\varphi_t|^2\,\mathrm{d}t < \infty\}$ and denote by $\mathcal{P}^2$ the subspace of $\mathcal{H}^2$ formed by the predictable processes;
- $\mathcal{S}^2 = \{(\varphi_t)_{0\leqslant t\leqslant T}\colon$ an $\mathcal{F}_t$-progressively measurable, real-valued, càdlàg process such that $E(\sup_{0\leqslant t\leqslant T} |\varphi(t)|^2) < \infty\}$;
- $l^2 = \{(x_i)_{i\geqslant 1}\colon$ a real-valued sequence such that $\sum_{i=1}^{\infty} x_i^2 < \infty\}$;
- $A^2 = \{(K_t)_{0\leqslant t\leqslant T}\colon$ an $\mathcal{F}_t$-adapted, continuous, increasing process such that $K_0 = 0, E|K_T|^2 < \infty\}$.

We shall denote by $\mathcal{H}^2(l^2)$ and $\mathcal{P}^2(l^2)$ the corresponding spaces of l^2-valued process equipped with the norm $\|\varphi\|^2 = \sum_{i=1}^{\infty} E\int_0^T |\varphi_t^{(i)}|^2\,\mathrm{d}t$.

We denote by $(H^{(i)})_{i\geqslant 1}$ the Teugels martingales associated with the Lévy process $\{L_t\colon t \in [0,T]\}$. More precisely

$$H_t^{(i)} = c_{i,i}Y_t^{(i)} + c_{i,i-1}Y_t^{(i-1)} + \cdots + c_{i,1}Y_t^{(1)},$$

where $Y_t^{(i)} = L_t^{(i)} - E[L_t^{(i)}] = L_t^{(i)} - tE[L_1^{(i)}]$ for all $i \geqslant 1$ and $L_t^{(i)}$ are power-jump processes. That is, $L_t^{(1)} = L_t$ and $L_t^{(i)} = \sum_{0<s\leqslant t} (\triangle L_t)^i$ for $i \geqslant 2$. For more details on Teugels martingales, one can see Nualart and Schoutens [6].

We consider the following assumptions:

(H1) The terminal value $\xi \in L^2(\Omega, \mathcal{F}_T, P)$.
(H2) The coefficients $f: [0,T] \times \Omega \times \mathbb{R} \times l^2 \to \mathbb{R}$ and $g: [0,T] \times \Omega \times \mathbb{R} \times l^2 \to \mathbb{R}$ are progressively measurable, such that $f(\cdot, 0, 0) \in \mathcal{H}^2$, $g(\cdot, 0, 0) \in \mathcal{H}^2$.

(H3) There exists some constants $C > 0$ and $0 < \alpha < 1$ such that for every $(\omega, t) \in \Omega \times [0, T], (y_1, z_1), (y_2, z_2) \in \mathbb{R} \times l^2$
$|f(t, y_1, z_1) - f(t, y_2, z_2)|^2 \leqslant C(|y_1 - y_2|^2 + \|z_1 - z_2\|^2)$, P-a.s.,
$|g(t, y_1, z_1) - g(t, y_2, z_2)|^2 \leqslant C|y_1 - y_2|^2 + \alpha\|z_1 - z_2\|^2$, P-a.s.

(H4) The obstacle process $(S_t)_{0\leqslant t\leqslant T}$, which is an $\mathcal{F}_t$-progressively measurable, real-valued, càdlàg process satisfying that $S_T \leqslant \xi$ a.s. and

$$E\Big[\sup_{0\leqslant t\leqslant T} \big(S_t^+\big)^2\Big] < +\infty; \quad \text{where } S_t^+ = \max\{S_t, 0\}.$$

Moreover, we assume that its jumping times are inaccessible stopping times [4].

Definition 1. A solution of Eq. (1) is a triple $(Y_t, Z_t, K_t)_{0\leqslant t\leqslant T}$ with values in $\mathbb{R} \times l^2 \times \mathbb{R}$ associated with (ξ, f, g, S) and satisfies that

$$\begin{cases} \text{(i)} \quad (Y_t, Z_t)_{0\leqslant t\leqslant T} \in \mathcal{S}^2 \times \mathcal{P}^2\big(l^2\big) \quad \text{and} \quad (K_t)_{0\leqslant t\leqslant T} \in A^2; \\ \text{(ii)} \quad Y_t = \xi + \int_t^T f(s, Y_{s-}, Z_s)\,\mathrm{d}s + \int_t^T g(s, Y_{s-}, Z_s)\,\mathrm{d}B_s + K_T - K_t - \sum_{i=1}^{\infty} \int_t^T Z_s^{(i)}\,\mathrm{d}H_s^{(i)}, \quad 0 \leqslant t \leqslant T, \text{ a.s.}; \\ \text{(iii)} \quad \text{for all } 0 \leqslant t \leqslant T, Y_t \geqslant S_t, \text{ a.s.}; \\ \text{(iv)} \quad \int_0^T (Y_{t-} - S_{t-})\,\mathrm{d}K_t = 0, \text{ a.s.} \end{cases} \tag{2}$$

3. The main results

Firstly, we consider the special case that is the function f and g do not depend on (Y, Z), i.e. $f(\omega, t, y, z) \equiv f(\omega, t)$, $g(\omega, t, y, z) \equiv g(\omega, t)$, for all $(t, y, z) \in [0, T] \times \mathbb{R} \times l^2$ via Snell envelope.

Theorem 2. *Assume that* (H1), $f \in \mathcal{H}^2$, $g \in \mathcal{H}^2$ *and* (H4) *hold. Then, there exists a triple* $(Y_t, Z_t, K_t)_{0\leqslant t\leqslant T}$ *solution of the RBDSDEs* (1) *associated with* (ξ, f, g, S).

Proof. We set the filtration $\{\mathcal{G}_t, t \in [0, T]\}$ by

$$\mathcal{G}_t = \mathcal{F}_{0,t}^L \vee \mathcal{F}_{0,T}^B. \tag{3}$$

For $f \in \mathcal{H}^2$, $g \in \mathcal{H}^2$, $\xi \in L^2(\Omega, \mathcal{F}_T, P)$, let $\eta = \{\eta_t\}_{0\leqslant t\leqslant T}$ be the process defined as follows:

$$\eta_t = \xi 1_{\{t=T\}} + S_t 1_{\{t<T\}} + \int_0^t f(s)\,\mathrm{d}s + \int_0^t g(s)\,\mathrm{d}B_s, \tag{4}$$

then when $t < T$, η is a càdlàg, $\mathcal{G}_t$-adapted process which has the same jump times as S. Moreover,

$$\sup_{0\leqslant t\leqslant T} |\eta_t| \in L^2(\Omega). \tag{5}$$

So, the Snell envelope of η is the smallest càdlàg supermartingale which dominates the process η and it is given by:

$$\mathcal{S}_t(\eta) = \operatorname{esssup}_{v\in\mathcal{T}} E[\eta_v|\mathcal{G}_t], \tag{6}$$

where $\mathcal{T}$ is the set of all $\mathcal{G}_t$-stopping time such that $0 \leqslant \tau \leqslant T$.

Due to (4), we have

$$E\Big[\sup_{0\leqslant t\leqslant T} \big|\mathcal{S}_t(\eta)\big|^2\Big] < +\infty, \tag{7}$$

and then $\{\mathcal{S}_t(\eta)\}_{0\leqslant t\leqslant T}$ is of class [D]. Hence, it has the following Doob–Meyer decomposition:

$$\mathcal{S}_t(\eta) = E\left[\xi + \int_0^T f(s)\,\mathrm{d}s + \int_0^T g(s)\,\mathrm{d}B_s + K_T|\mathcal{G}_t\right] - K_t, \tag{8}$$

where $\{K_t\}_{0\leqslant t\leqslant T}$ is a $\mathcal{G}_t$-adapted càdlàg, non-decreasing process such that $K_0=0$. From [2], we have $E[K_T]^2<+\infty$. It follows that

$$E\Big[\sup_{0\leqslant t\leqslant T}\Big|E\Big(\xi+\int_0^T f(s)\,\mathrm{d}s+\int_0^T g(s)\,\mathrm{d}B_s+K_T|\mathcal{G}_t\Big)\Big|^2\Big]<+\infty. \tag{9}$$

Predictable representation property [6] yields that there exists $Z\in\mathcal{P}^2(l^2)$ such that

$$\begin{aligned}M_t &\triangleq E\Big[\xi+\int_0^T f(s)\,\mathrm{d}s+\int_0^T g(s)\,\mathrm{d}B_s+K_T|\mathcal{G}_t\Big]\\ &=E\Big[\xi+\int_0^T f(s)\,\mathrm{d}s+\int_0^T g(s)\,\mathrm{d}B_s+K_T\Big]+\sum_{i=1}^{\infty}\int_0^t Z_s^{(i)}\,\mathrm{d}H_s^{(i)}.\end{aligned} \tag{10}$$

From the property of Lévy process, we know that M is quasi-left-continuous. So, M has only inaccessible jump times.

Now, we show that K is a continuous process. From [2], we know that the jump times of K is included in the set $\{\triangle K\neq 0\}\subset\{\mathcal{S}_-(\eta)=\eta_-\}$ where η_- is the left limit process.

Now let τ be a predictable time, then:

$$E\big[\mathcal{S}_{\tau-}(\eta)1_{\{\triangle K_\tau>0\}}\big]=E[\eta_{\tau-}1_{\{\triangle K_\tau>0\}}]\leqslant E[\eta_\tau 1_{\{\triangle K_\tau>0\}}]\leqslant E\big[\mathcal{S}_\tau(\eta)1_{\{\triangle K_\tau>0\}}\big]. \tag{11}$$

The first inequality is obtained through the fact that the process η has inaccessible jumping times, and may have a positive jump at T.

On the other hand,

$$\begin{aligned}E\big[\mathcal{S}_{\tau-}(\eta)1_{\{\triangle K_\tau=0\}}\big]&=E\big[(M_{\tau-}+K_\tau)1_{\{\triangle K_\tau=0\}}\big]=E\big[(M_\tau+K_\tau)1_{\{\triangle K_\tau=0\}}\big]\\ &=E\big[\mathcal{S}_\tau(\eta)1_{\{\triangle K_\tau=0\}}\big].\end{aligned} \tag{12}$$

Then from (11) and (12), we have $E[\mathcal{S}_{\tau-}(\eta)]\leqslant E[\mathcal{S}_\tau(\eta)]$. Since $\mathcal{S}(\eta)$ is a supermartingale. For any predictable time τ, we have $E[\mathcal{S}_{\tau-}(\eta)]=E[\mathcal{S}_\tau(\eta)]$. So, $\{\mathcal{S}_t(\eta)\}_{0\leqslant t\leqslant T}$ is regular ([4], Definition 5.49), i.e. $\mathcal{S}_-(\eta)={}^p\,\mathcal{S}(\eta)$. Then, the process K is continuous ([4], Theorem 5.49).

Now let us set

$$Y_t=\operatorname{esssup}_{v\in\mathcal{T}_t}E\Big[\xi 1_{\{v=T\}}+S_v 1_{\{v<T\}}+\int_t^v f(s)\,\mathrm{d}s+\int_t^v g(s)\,\mathrm{d}B_s|\mathcal{G}_t\Big]. \tag{13}$$

Then

$$Y_t+\int_0^t f(s)\,\mathrm{d}s+\int_0^t g(s)\,\mathrm{d}B_s=\mathcal{S}_t(\eta)=M_t-K_t. \tag{14}$$

Henceforth, we have

$$Y_t+\int_0^t f(s)\,\mathrm{d}s+\int_0^t g(s)\,\mathrm{d}B_s=E\Big[\xi+\int_0^T f(s)\,\mathrm{d}s+\int_0^T g(s)\,\mathrm{d}B_s+K_T\Big]+\sum_{i=0}^{\infty}\int_0^t Z_s^{(i)}\,\mathrm{d}H_s^{(i)}-K_t. \tag{15}$$

So,

$$Y_t=\xi+\int_t^T f(s)\,\mathrm{d}s+\int_t^T g(s)\,\mathrm{d}B_s+K_T-K_t-\sum_{i=0}^{\infty}\int_t^T Z_s^{(i)}\,\mathrm{d}H_s^{(i)},\quad 0\leqslant t\leqslant T. \tag{16}$$

Since $Y_t+\int_0^t g(s)\,\mathrm{d}s=\mathcal{S}_t(\eta)$ and $\mathcal{S}_t(\eta)\geqslant\eta_t=\xi 1_{\{t=T\}}+S_t 1_{\{t<T\}}+\int_0^t f(s)\,\mathrm{d}s+\int_0^t g(s)\,\mathrm{d}B_s$. Then, for all $0\leqslant t\leqslant T$, we have $Y_t\geqslant S_t$.

Finally, from [2], we get $\int_0^T(\mathcal{S}_{t-}(\eta)-\eta_{t-})\,\mathrm{d}K_t=0$, i.e.

$$\int_0^T(Y_{t-}-S_{t-})\,\mathrm{d}K_t=\int_0^T\big(\mathcal{S}_{t-}(\eta)-\eta_{t-}\big)\,\mathrm{d}K_t=0. \tag{17}$$

So, the process $(Y_t,Z_t,K_t)_{0\leqslant t\leqslant T}$ is a solution of the RBDSDEs (1) associated with (ξ,f,g,S). □

Theorem 3. *Assume the assumptions* (H1)–(H4) *hold. Then, RBDSDEs* (1) *associated with* (ξ, f, g, S) *has a unique solution* $(Y_t, Z_t, K_t)_{0\leqslant t\leqslant T}$.

Proof. Let $\mathcal{H} = S^2 \times \mathcal{P}^2(l^2)$ endowed with the norm

$$\|(Y,Z)\|_\beta = \left(E\left[\int_0^T e^{\beta s}\left(|Y_{s-}|^2 + \sum_{i=1}^{\infty}|Z_s^{(i)}|^2\right)ds\right]\right)^{1/2}, \tag{18}$$

for a suitable constant $\beta > 0$. Let Φ be the map from $\mathcal{H}$ into itself and let $(\widetilde{Y}, \widetilde{Z})$ and $(\widetilde{Y}', \widetilde{Z}')$ be two elements of $\mathcal{H}$. Set

$$(Y,Z) = \Phi(\widetilde{Y}, \widetilde{Z}), \qquad (Y',Z') = \Phi(\widetilde{Y}', \widetilde{Z}'), \tag{19}$$

where (Y,Z,K) $((Y',Z',K'))$ is the solution of the RBDSDE associated with $(\xi, f(t,\widetilde{Y}_{t-},\widetilde{Z}_t), g(t,\widetilde{Y}_{t-},\widetilde{Z}_t), S)$ $((\xi, f(t,\widetilde{Y'_{t-}}, \widetilde{Z'_{t-}}), g(t,\widetilde{Y'_{t-}}, \widetilde{Z'_{t-}}), S))$.

By the Itô formula and integration by parts, we obtain

$$\begin{aligned}
e^{\beta t}(Y_t - Y_t')^2 &= -\beta\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})^2\,ds + 2\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})\big[f(s,\widetilde{Y}_{s-},\widetilde{Z}_s) - f(s,\widetilde{Y'_{s-}},\widetilde{Z'_{s-}})\big]ds\\
&\quad + 2\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})\big[g(s,\widetilde{Y}_{s-},\widetilde{Z}_s) - g(s,\widetilde{Y'_{s-}},\widetilde{Z'_{s-}})\big]dB_s\\
&\quad + 2\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})(dK_s - dK'_s) + \int_t^T e^{\beta s}\big|g(s,\widetilde{Y}_{s-},\widetilde{Z}_s) - g(s,\widetilde{Y'_{s-}},\widetilde{Z'_{s-}})\big|^2 ds\\
&\quad - 2\sum_{i=1}^{\infty}\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})(Z_s^{(i)} - Z_s'^{(i)})\,dH_s^{(i)}\\
&\quad - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\int_0^t e^{\beta s}(Z_s^{(i)} - Z_s'^{(i)})(Z_s^{(j)} - Z_s'^{(j)})\,d\big[H^{(i)},H^{(j)}\big]_s.
\end{aligned} \tag{20}$$

Noting that $\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})(dK_s - dK'_s) \leqslant 0$, using the fact $\langle H^{(i)}, H^{(j)}\rangle_t = \delta_{ij}t$ and taking the expectation on the both sides of (20), we obtain

$$\begin{aligned}
&E\big[e^{\beta t}(Y_t - Y_t')^2\big] + \beta E\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})^2\,ds + E\int_t^T e^{\beta s}\|Z_s - Z_s'\|^2\,ds\\
&\quad\leqslant \frac{2C}{1-\alpha}E\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})^2\,ds + \left(C + \frac{1-\alpha}{2}\right)E\int_t^T e^{\beta s}\big|\widetilde{Y}_{s-} - \widetilde{Y'_{s-}}\big|^2 ds\\
&\qquad + \frac{1+\alpha}{2}E\int_t^T e^{\beta s}\big\|\widetilde{Z}_{s-} - \widetilde{Z'_{s-}}\big\|^2 ds.
\end{aligned}$$

Let $\gamma = \frac{2C}{1-\alpha}$, $\bar{C} = 2(C + \frac{1-\alpha}{2})/1+\alpha$ and $\beta = \gamma + \bar{C}$, we get

$$\begin{aligned}
&E\big[e^{\beta t}|Y_t - Y_t'|^2\big] + \bar{C}E\int_t^T e^{\beta s}(Y_{s-}-Y'_{s-})^2\,ds + E\int_t^T e^{\beta s}\|Z_s - Z_s'\|^2\,ds\\
&\quad\leqslant \frac{1+\alpha}{2}E\int_t^T e^{\beta s}\big(\bar{C}\big|\widetilde{Y}_{s-} - \widetilde{Y'_{s-}}\big|^2 + \big\|\widetilde{Z}_{s-} - \widetilde{Z'_{s-}}\big\|^2\big)ds.
\end{aligned}$$

Noting that $E[e^{\beta t}(Y_t - Y_t')^2] \geqslant 0$, we obtain

$$E\int_t^T e^{\beta s}\big(\bar{C}\big|Y_{s-}-Y'_{s-}\big|^2\,ds+\big\|Z_s-Z'_s\big\|^2\big)\,ds\leqslant\frac{1+\alpha}{2}E\int_t^T e^{\beta s}\big(\bar{C}\big|\widetilde{Y}_{s-}-\widetilde{Y'_{s-}}\big|^2+\big\|\widetilde{Z}_{s-}-\widetilde{Z'_{s-}}\big\|^2\big)\,ds,$$

that is $\|(Y,Z)\|_\beta^2\leqslant\frac{1+\alpha}{2}\|(Y',Z')\|_\beta^2$. From which it follows that Φ is a strict contraction on $\mathcal{H}$ with the norm $\|\cdot\|_\beta$ where β is defined as above. Then, Φ has a unique fixed point $(Y,Z)\in\mathcal{H}$, from the Burkholder–Davis–Gundy inequality, which is the unique solution of RBDSDEs (1). $\square$

Acknowledgement

The author would like to thank the anonymous referee for the valuable comments and correcting some errors.

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