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Partial Differential Equations

A Note on the analytic solutions of the Camassa–Holm equation ☆

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Abstract

In this Note we are concerned with the well-posedness of the Camassa–Holm equation in analytic function spaces. Using the Abstract Cauchy–Kowalewski Theorem we prove that the Camassa–Holm equation admits, locally in time, a unique analytic solution. Moreover, if the initial data is real analytic, belongs to $H^s(\mathbb{R})$ with $s > 3/2$, $\|u_0\|_{L^1} < \infty$ and $u_0 - u_{0xx}$ does not change sign, we prove that the solution stays analytic globally in time. ***To cite this article: M.C. Lombardo et al., C. R. Acad. Sci. Paris, Ser. I 341 (2005).***

Résumé

Solutions analytiques de l'équation de Camassa–Holm. On étude ici l'équation de Camassa–Holm dans les espaces des fonctions analytiques. On montre que, si les données initiales sont analytiques, il existe, localement dans le temp, une solution unique analytique.

En outre si la la donnée initiale analytique $u_0(x)$ est bornée dans L^1, appartient à $H^s(\mathbb{R})$ $s > 3/2$ et satisfait la condition $u_0 - u_{0xx} \geqslant 0$, la solution résulte analytique globalement dans le temp. ***Pour citer cet article : M.C. Lombardo et al., C. R. Acad. Sci. Paris, Ser. I 341 (2005).***

Version française abrégée

Dans cette Note nous montrons que l'equation de Camassa–Holm (1) avec des données initiales analytique a, pour un temp dependant des données, une unique solution analytique. Notre preuve est basée sur le Théorème de Cauchy–Kowalewski dans sa formulation abstraite, voir le Théorème 2.5. En effet on peut prouver que l'equation de Camassa–Holm, écrite dans sa forme pseudo-différentielle (5), satisfait les hypotheses du Théorème de Cauchy–Kowalewski (l'opérateure est quasi contractif dans les normes analytiques). Cette propriété est une directe conséquence de l'estimation de Cauchy pour la dérivée d'une fonction analytique. Donc, notre premier résultat est le suivant :

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Théorème 0.1. *On suppose que la donnée initiale de l'equation de Camassa–Holm* (1) $u_0 \in H^{1,\rho_0}$ *avec* $\rho_0 > 0$. *Alors ils existent* $\rho > 0$, $\beta > 0$ *et* $T > 0$ *telles que l'Éq.* (1) *admet une unique solution* $u \in H^{1,\rho}_{\beta,T}$.

Notre deuxième résultat concerne l'existence globale dans le temp des solutions analytiques. Il est bien connu (voire, e.g., [17]) que, si la donnée initiale est tel que le potentiel $(1 - \partial_x^2)u_0 \geqslant 0$ (ou $\leqslant 0$), la solution de l'equation de Camassa–Holm est globale. Nous montrons le même résultat pour les solutions analytiques réelles.

Théorème 0.2. *On suppose que la donnée initiale de l'equation de Camassa–Holm* (1) $u_0 \in H^{1,\rho}$ *avec* $\rho > 0$. *En plus on suppose que* $u_0 \in L^1$ *et que* $(1 - \partial_x^2)u_0 \geqslant 0$ (*ou* $\leqslant 0$). *Alors pour tout* $T > 0$ *il existe une solution unique analytique réelle* $u(x,t)$ *avec* $t \in [0,T]$.

La preuve du théorème est basée sur la construction des approximations de Galerkin de la solution, et sur une estimation a priori globale de la solution dans les espaces $H^{r,\rho}$. L'estimation a priori est dans (14) et est basée sur les estimations du Lemma 3.2.

Le sens de l'estimation a priori est que la region du plane complexe où la solution est analytique se réduit avec un taux exponential et donc il y a toujours une bande d'analyticité de la solution.

1. Introduction

In recent years much attention has been devoted to the study of the Camassa–Holm equation (CH), namely:

$$u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \tag{1}$$

where $u = u(x,t)$ with $x \in \mathbb{R}$ and $t \geqslant 0$. Eq. (1) was first introduced by Fokas and Fuchssteiner [11] as part of the study of some general aspects of soliton theory and later Camassa and Holm gave a physical derivation of it [1].

During the past seven years there has been an extensive investigation about the problem of the well-posedness of the Camassa–Holm equation. Local well-posedness of classical solutions in the Sobolev space H^3 was first proved in [5] and [4]. The result was later improved for initial data in H^s with $s > 3/2$ through different methods, both in the periodic and non periodic cases (see for example [15,17,7,12,16,8] and references therein).

The existence of global weak solutions was obtained in [6] with initial data $u_0 \in H^1(\mathbb{R})$ such that $u_0 - u_{0xx}$ is a positive regular Borel measure on $\mathbb{R}$ with total bounded variation.

Global well-posedness of classical solutions to the Camassa–Holm equation was proved by Rodríguez-Blanco in [17].

On the other hand it is well known that, under proper conditions, the solutions of the CH equations can develop a singularity in finite time [1,2,5,3].

In this Note first we shall prove that, if the initial datum u_0 is analytic then the solution stays analytic for a short time:

Theorem 1.1. *Let* $u_0 \in H^{1,\rho_0}$ *be the initial datum of the Camassa–Holm equation* (1). *Then there exists* $\beta > 0$ *such that for any* ρ *with* $0 < \rho < \rho_0$ *there exists a unique continuously differentiable* (*w.r.t. time*) *solution* u *of the Camassa–Holm equation with the following property*:

- $u(\cdot,t) \in H^{1,\rho}$ *and* $\partial_t u(\cdot,t) \in H^{1,\rho}$ *when* $t \in [0, \frac{\rho_0 - \rho}{\beta}]$.

For the definition of the function spaces H^{1,ρ_0} see Section 2 below. Roughly speaking H^{1,ρ_0} is the space of functions $f(x)$ analytic w.r.t x in a strip of width ρ_0 in the complex plane $\mathbb{C}$. The space $H^{1,\rho}_{\beta,T}$ is the space of functions $f(x,t)$, analytic w.r.t. x in a strip of the complex plane whose width shrinks linearly in time at speed β. The above theorem says the solution $u \in H^{1,\rho}_{\beta,T}$. The proof of the above theorem is based on the abstract Cauchy–Kowalewski Theorem in Banach spaces in the form given by Safonov in [18].

A similar result (for the CH equation in periodic domain) appeared in [13].

Secondly, we state a global-in-time result. If the initial data belongs to $H^s(\mathbb{R})$ with $s > 3/2$ and $u_0 - \partial_x^2 u_0$ does not change sign, we prove that time analyticity of the solutions to the Camassa–Holm equation holds globally. Using some a priori estimates and the result of global well-posedness given in Theorem 3.1, we are able to prove that the solution is analytic, globally in time, with values in a Gevrey class of functions. Our result is the following:

Theorem 1.2. *Suppose the initial datum of the Camassa–Holm equation satisfy* $u_0 \in H^{r,\rho}$ *with* $r > 3/2$, $|u_0|_{L^1} < \infty$, $u_0 - \partial_x^2 u_0 \geqslant 0$ (*or* $\leqslant 0$). *Then the unique solution* $u(x,t)$ *belongs to the Gevrey class of index* 1 *globally in time.*

In the proof of global analyticity of the solutions we shall closely follow the ideas of [14,10,9].

In this Note we consider the CH equation in $\mathbb{R}$. With minor modification one can obtain the same results for the CH equation in a periodic domain.

2. Local in time analyticity

In the sequel we shall be dealing with functions that are analytic in the complex variable x. We hence introduce the strip in the complex plane.

$$D(\rho) = \mathbb{R} \times (-\rho, \rho) = \{x \in \mathbb{C}\colon\ \Im x \in (-\rho, \rho)\}.$$

The L^2 integration is performed along the following path:

$$\Gamma(b) = \{x \in \mathbb{C}\colon\ \Im x = b\}.$$

We can now introduce the function spaces where local in time analyticity of the CH equation will be proved. Let use first introduce the function spaces of the initial datum.

Definition 2.1. $H^{0,\rho}$ is the set of all complex functions $f(x)$ such that

- f is analytic in $D(\rho)$;
- $f \in L^2(\Gamma(\Im x))$ for $\Im x \in (-\rho, \rho)$; i.e. if $\Im x$ is inside $(-\rho, \rho)$, then $f(\Re x + \mathrm{i}\Im x)$ is a square integrable function of $\Re x$;
- $|f|_\rho = \sup_{\Im x \in (-\rho,\rho)} \|f(\cdot + \mathrm{i}\Im x)\|_{L^2(\Gamma(\Im x))} < \infty$.

Definition 2.2. $H^{k,\rho}$ is the set of all complex functions $f(x)$ such that

- $\partial_x^j f \in H^{0,\rho}$ for $0 \leqslant j \leqslant k$;
- $\|f\|_{k,\rho} \equiv \sum_{0 \leqslant j \leqslant k} |\partial_x^j f|_\rho < \infty$.

This norm is equivalent to:

$$\|f\|_{k,\rho} = \left[\int \mathrm{e}^{2\rho|\xi|}\left(1 + |\xi|^2\right)^k \left|\hat{f}(\xi)\right|^2 \mathrm{d}\xi\right]^{1/2}, \tag{2}$$

where $\hat{f}(\xi)$ if the Fourier transform of f. With the above expression of the norm one can define $H^{r,\rho}$ with $r \in \mathbb{R}^+$. This will be useful in Section 3.

We now introduce the dependence on the time variable.

Definition 2.3. The space $H^{1,\rho}_{\beta,T}$ is defined as the space of all the functions $f(x,t)$ such that:

- $f \in C^1([0,T], H^{1,\rho})$.
- $|f|_{1,\rho,\beta,T} = \sup_{0\leqslant t\leqslant T} |f(t,\cdot)|_{0,\rho-\beta t} + \sup_{0\leqslant t\leqslant T} |\partial_t f(t,\cdot)|_{0,\rho-\beta t} + \sup_{0\leqslant t\leqslant T} |\partial_x f(t,\cdot)|_{0,\rho-\beta t} < \infty$.

One can analogously define the space $H^{k,\rho}_{\beta,T}$. We shall also need the following estimate which is a direct consequence of the Cauchy estimate on the derivative of analytic functions:

Lemma 2.4. *Let $u, w \in H^{1,\rho''}$. If $\rho' < \rho''$ then*

$$\|u\partial_x u - w\partial_x w\|_{1,\rho'} \leqslant c\frac{\|u-w\|_{1,\rho''}}{\rho''-\rho'}. \tag{3}$$

We now write the Camassa–Holm in a form suitable for the application of the ACK Theorem. It is easy to see that the CH can be written in the form:

$$\bigl(1-\partial_x^2\bigr)(u_t+uu_x) = -2uu_x - u_x u_{xx} = -\partial_x\left(u^2+\frac{1}{2}u_x^2\right).$$

Thus one can write the CH equation in the pseudodifferential form:

$$u_t + uu_x = -\frac{\mathrm{i}\xi}{1+\xi^2}\left(u^2+\frac{1}{2}u_x^2\right), \tag{4}$$

where ξ is the dual Fourier variable of x. With an integration in time one gets:

$$u = F(t,u) \quad \text{where } F(t,u) \equiv u_0 - \int_0^t \mathrm{d}t'\left[uu_x + \frac{\mathrm{i}\xi}{1+\xi^2}\left(u^2+\frac{1}{2}u_x^2\right)\right]. \tag{5}$$

We now state the ACK Theorem in the form given by Safonov in [18]. Consider the problem:

$$u = F(t,u). \tag{6}$$

Theorem 2.5 (ACK). *Suppose that $\exists R > 0$, $\rho_0 > 0$, and $\beta_0 > 0$ such that if $0 < t \leqslant \rho_0/\beta_0$, the following properties hold*:

(a) $\forall 0 < \rho' < \rho \leqslant \rho_0$ *and* $\forall u$ *s.t.* $\{u \in X_\rho\colon \sup_{0\leqslant t\leqslant T}|u(t)|_\rho \leqslant R\}$ *the map* $F(t,u)\colon[0,T] \mapsto X_{\rho'}$ *is continuous.*
(b) $\forall 0 < \rho < \rho_0$ *the function* $F(t,0)\colon[0,\rho_0/\beta_0] \mapsto \{u \in X_\rho\colon \sup_{0\leqslant t\leqslant T}|u(t)|_\rho \leqslant R\}$ *is continuous and*

$$\bigl|F(t,0)\bigr|_\rho \leqslant R_0 < R. \tag{7}$$

(c) $\forall 0 < \rho' < \rho(s) < \rho_0$ *and* $\forall u$ *and* $w \in \{u \in X_\rho\colon \sup_{0\leqslant t\leqslant T}|u(t)|_{\rho-\beta_0 t} \leqslant R\}$,

$$\bigl|F(t,u)-F(t,w)\bigr|_{\rho'} \leqslant C\int_0^t \mathrm{d}s\left(\frac{|u-w|_{\rho(s)}}{\rho(s)-\rho'}\right). \tag{8}$$

Then $\exists \beta > \beta_0$ such that $\forall 0 < \rho < \rho_0$ Eq. (6) *has a unique solution $u(t) \in X_\rho$ with $t \in [0,(\rho_0-\rho)/\beta]$; moreover $\sup_{\rho<\rho_0-\beta t}|u(t)|_\rho \leqslant R$.*

The proof of the short time existence of analytic solutions of the CH equation easily follows as a simple application of the ACK Theorem. In fact it is obvious that the operator $F(u,t)$ given by (5) satisfies the hypotheses (a) and (b) of the ACK Theorem. Regarding hypothesis (c), considering first the norm $|F(u,t)|_{1,\rho'}$, the bound is easily achieved by using Lemma 2.4. Finally, the bound on $|\partial_x F(u,t)|_{1,\rho'}$ is obtained using the fact that the operator $\xi^2/(1+\xi^2)$ is bounded.

The proof of Theorem 1.1 (or the equivalent form Theorem 0.1) is thus achieved.

3. Global in time analyticity

The function space we shall deal with in this section is the space $H^{r,\rho}$, $r \in \mathbb{R}^+$, $r > 3/2$, endowed with the norm (2). One can see [14] that the space G^1 of Gevrey functions of index 1 can be recovered as:

$$G^1 = \bigcup_{\rho>0} H^{r,\rho}.$$

Here we state a global existence theorem in Sobolev spaces [17], that will be useful in the sequel:

Theorem 3.1 (Rodríguez-Blanco). *Suppose that $u_0 \in H^s(\mathbb{R})$, $s > 3/2$, with $(1-\partial_x^2)u_0 \geqslant 0$ and that $u_0 \in L^1(\mathbb{R})$. Then Eq.* (1) *is globally well posed in $H^s(\mathbb{R})$.*

To prove our result, Theorem 1.2, the crucial step is the global estimate on u in the space $H^{r,\rho}$, with ρ depending on time t. We will show that, if ρ decreases to 0 exponentially fast with the appropriate rate (see (13) below), the norm of u in $H^{r,\rho}$ is bounded in $[0,T]$ for any T. To simplify the notation we shall formally derive the estimate for u but all the computations can be made rigorous by considering a Galerkin approximation.

Consider the solution u of the CH equation (4). Using the usual notation for the scalar product in L^2, and denoting with $\|\cdot\|$ and $\|\cdot\|_{H^r}$ the norms in L^2 and H^r, one can write:

$$\begin{aligned}\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\big\||\xi|^r\,\mathrm{e}^{\rho|\xi|}u\big\|_{L^2}^2 &= \dot\rho\big\langle|\xi|^{r+1}\,\mathrm{e}^{\rho|\xi|}u,|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\big\rangle+\big\langle|\xi|^r\,\mathrm{e}^{\rho|\xi|}\partial_t u,|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\big\rangle\\ &=\dot\rho\big\||\xi|^{r+1/2}\,\mathrm{e}^{\rho|\xi|}u\big\|^2-\big\langle|\xi|^r\,\mathrm{e}^{\rho|\xi|}uu_x,|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\big\rangle\\ &\quad-\left\langle|\xi|^r\,\mathrm{e}^{\rho|\xi|}\frac{\mathrm{i}\xi}{1+\xi^2}\left(u^2+\frac{u_x^2}{2}\right),|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\right\rangle.\end{aligned}\tag{9}$$

Notice how in the above calculation we have taken ρ (i.e. the rate of decay of the spectrum or, equivalently, the width of the strip of analyticity) as a function of time. Now we state our basic estimates:

Lemma 3.2. *Let $u \in \mathcal{D}(|\xi|^{r+1/2}\,\mathrm{e}^{\rho|\xi|})$ with $r > 3/2$, then*:

$$\big|\big\langle|\xi|^r\,\mathrm{e}^{\rho|\xi|}uu_x,|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\big\rangle\big|\leqslant c\|u\|_{H^r}\big(\|u\|_{H^r}^2+2\rho\big\||\xi|^{r+1/2}\,\mathrm{e}^{\rho|\xi|}u\big\|^2\big),\tag{10}$$

$$\left|\left\langle|\xi|^r\,\mathrm{e}^{\rho|\xi|}\frac{\mathrm{i}\xi}{1+\xi^2}\left(u^2+\frac{u_x^2}{2}\right),|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\right\rangle\right|\leqslant c\|u\|_{H^r}\big(\|u\|_{H^r}^2+2\rho\big\||\xi|^{r+1/2}\,\mathrm{e}^{\rho|\xi|}u\big\|^2\big).\tag{11}$$

Proof. The proof of the estimate (10) is standard. In estimate (11) the most difficult term is the term involving the derivative of u. The proof of the estimate is the following:

$$\begin{aligned}\left|\left\langle|\xi|^r\,\mathrm{e}^{\rho|\xi|}\frac{\mathrm{i}\xi}{1+\xi^2}u_x^2,|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\right\rangle\right| &\leqslant \big|\big\langle|\xi|^{r-1}\,\mathrm{e}^{\rho|\xi|}u_x^2,|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\big\rangle\big|\leqslant\|u_x\|_{L^\infty}\big|\big\langle|\xi|^{r-1}\,\mathrm{e}^{\rho|\xi|}u_x,|\xi|^r\,\mathrm{e}^{\rho|\xi|}u\big\rangle\big|\\ &=\|u_x\|_{L^\infty}\int_{\mathbb{R}}|\xi|^{2r}\,\mathrm{e}^{2\rho|\xi|}\big|\hat u(\xi)\big|^2\,\mathrm{d}\xi\\ &\leqslant\|u_x\|_{L^\infty}\int_{\mathbb{R}}|\xi|^{2r}\big(1+2\rho|\xi|\,\mathrm{e}^{2\rho|\xi|}\big)\big|\hat u(\xi)\big|^2\,\mathrm{d}\xi\\ &\leqslant c_1\|u\|_{H^r}\big(\big\||\xi|^r u\big\|^2+2\rho\big\||\xi|^{r+1/2}\,\mathrm{e}^{\rho|\xi|}u\big\|^2\big)\\ &\leqslant c_1\|u\|_{H^r}\big(\|u\|_{H^r}^2+2\rho\big\||\xi|^{r+1/2}\,\mathrm{e}^{\rho|\xi|}u\big\|^2\big).\end{aligned}$$

In the fourth inequality we have used the fact that $\mathrm{e}^y \leqslant 1 + y\,\mathrm{e}^y$ for $y \geqslant 0$ while in the fifth inequality we have used the Sobolev estimate on $\|u_x\|_{L^\infty}$. $\square$

Using Lemma 3.2 in (9) we get:

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\big\||\xi|^r\,\mathrm{e}^{\rho|\xi|}u\big\|^2\leqslant c\big(\dot\rho+4\rho\|u\|_{H^r}\big)\big\||\xi|^{r+1/2}\,\mathrm{e}^{\rho|\xi|}u\big\|^2+2c\|u\|_{H^r}^3.\tag{12}$$

By virtue of the proof of Theorem 3.1 (Section 4 in [17]), which guarantees the existence of a unique global solution, there exists a function $\theta(t)$, with θ^3 integrable in $[0,T]$ for each $T>0$, such that $\|u\|_{H^r}\leqslant\theta(t)$, $\forall t\in[0,\infty)$. So, provided that:

$$\dot\rho+6\rho\|u\|_{H^r}\leqslant 0,\quad\text{which is insured by}\quad\rho(t)\leqslant\rho(0)\exp\big(-4\|u\|_{H^r}t\big),\tag{13}$$

we get the bound in $t \in [0, T]$:

$$\left\| |\xi|^r \mathrm{e}^{\rho|\xi|} u \right\|^2 \leqslant \left\| |\xi|^r \mathrm{e}^{\rho(0)|\xi|} u_0 \right\|^2 + 2c \int_0^t \theta^3(t')\,\mathrm{d}t'. \tag{14}$$

We stress once again that such a bound can be obtained for any $T > 0$. The rigorous construction of global analytic solutions is now standard and the proof of Theorem 1.2 is achieved. The details will appear in a forthcoming paper.

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